ISSN 1000-9825, CODEN RUXUEW
Journal of Software, Vol.17, No.5, May 2006, pp.1133−1139
DOI: 10.1360/jos171133

E-mail: jos@iscas.ac.cn
http://www.jos.org.cn
Tel/Fax: +86-10-62562563

一种同时纠正量子随机错误和量子突发错误算法*

郭 迎[1+], 曾贵华[1], 马少军[2]

[1](上海交通大学 电子工程系,上海 200030)
[2](莱阳农学院 理学院,山东 青岛 266109)

How to Correct Random and Burst Quantum Errors Simultaneously

GUO Ying[1+], ZENG Gui-Hua[1], MA Shao-Jun[2]

[1](Department of Electronic Engineering, Shanghai Jiaotong University, Shanghai 200030, China)
[2](Science of School, Laiyang Agriculture College, Qingdao 266109, China)
+ Corresponding author: Phn: +86-21-62932028, Fax: +86-21-62933189, E-mail: yingguo1001@sjtu.edu.cn

Guo Y, Zeng GH, Ma SJ. How to correct random and burst quantum errors simultaneously. *Journal of Software*, 2006,17(5):1133−1139. http://www.jos.org.cn/1000-9825/17/1133.htm

Abstract: A special quantum error correction scheme is proposed to protect the flow of transmitted quantum information in complex channels. Based on the derived syndrome, an algorithm is devised for the construction of the called quantum event-error correction code, which can correct simultaneously random and burst quantum errors. Moreover, the constructed code can detect the length, the number and even the exact location of the occurring errors in the listed error event.
Key words: quantum event error; stabilizer; CSS code; quantum error correction code; quantum information

摘 要: 为了同时检测量子随机错误和量子突发错误,提出了量子事件错误检错码.通过利用构造的错误图样,该码不但检测并纠正错误发生的事件类型,而且可以检测到错误发生的种类、随机错误的数量、错误发生的长度甚至错误发生的位置.
关键词: 量子事件错误;稳定子;CSS 码;量子纠错码;量子信息
中图法分类号: O157 文献标识码: A

Quantum information has stimulated much interest with rapid development of quantum communication and quantum computation. An important issue in quantum information is quantum error prevention[1], detection[2] and correction[3,4]. So, the quantum error-correcting code is now an active area of research[5]. Since the pioneer investigations were proposed to defend decoherence in entangled states[6,7], many works considering to construct the quantum error correction codes(QECC) have been presented[8,9]. These codes, which are means of storing information in a certain set of qubits in such a way that it can be extracted even though a subset of the qubits have been changed in an unknown way, are fundamental parts in the investigation of the quantum information and quantum computing. It is an optimal candidate in quantum information for quantum error-correction, which

* Supported by the National Natural Science Foundation of China under Grant No.60472018 (国家自然科学基金)
Received 2005-03-19; Accepted 2005-07-08

compensates effects of quantum noise by introducing redundancy like the classical error correction code.

While the possibility of correcting decoherence errors in entangled states was discovered[6,7], many works considering QECC have been presented[8−13]. All these investigations may be divided into two categories, i.e., one is to construct codes for correcting quantum random errors[8−11] and the other is to design codes for defending quantum burst errors[12,13]. In the first category, errors happen in some random positions. To correct this kind of errors, the quantum error-correction code is designed to maximize the minimum number of corrected singular error symbols. Currently, such a category of errors has been intensively investigated, and hence many good quantum error-correction codes have been proposed with big error-correcting ability, which are decided by the weight of the constructed code. However, in the second category, errors happen in consecutive qubits with a fixed error length d. To correct this kind of errors, the burst error correction code is proposed to maximize the minimum length of corrected errors, which has a character of correcting errors with a fraction of fixed length.

While random errors and burst errors occur simultaneously in a quantum code, detection and correction of errors are more complex. A simple way is to divide these errors into M separated (single) error events. Each event can be described by a binary vector in which the first and last bits are always “1”s. Since there are three basic errors, i.e., the bit flip error, the phase error and the mixed error of bit flip and phase errors[14] occurring possibly to a qubit, quantum error events may also be divided into three kinds, i.e., quantum bit flip error event, quantum phase error event and quantum mixed error event. By far, there is no research on how to correct quantum event error by the quantum approach. To correct quantum event errors in a quantum code, a new code which is called as quantum event-error correcting (QED) code is first investigated in this paper.

1 Descriptions of Quantum Event Errors

In a two-dimension Hilbert space $H^{\otimes 2}$, a qubit can be denoted by $|\Psi\rangle=\alpha|0\rangle+\beta|1\rangle$. Since disturbance of the environment, errors may occur to such a qubit. Let ε be an arbitrary quantum error with error operation in $\{E_i:i=1,2,\ldots,n\}$ occurring to the qubit $|\psi\rangle$, where

$$E_i=\hat{e}_0 I+\hat{e}_1 X+\hat{e}_2 Z+\hat{e}_3 Y \tag{1}$$

$\hat{e}_\kappa(\kappa=0,1,2,3)$ are unit vectors, and operators $X=\sigma_x, Z=\sigma_z$ and $Y=-iXZ$ denote the bit flip error, the phase error and the mixed error of bit flip and phase error.

The quantum bit-flip error event (QBEE) can be described as,

$$e_x=(a_1,a_2,\ldots,a_{l(e_x)}), \tag{2}$$

where $a_1=a_{l(e_x)}$ and $a_i\in\{0,1\}$ for $1<k<l(e_x)$, and $l(e_x)$ denotes the length of e_x. If $a_i=1$, the corresponding qubit subjects to X error, otherwise there is no error occurring to the i-th qubit. Suppose there is a QBEE occurring to a quantum code, the corresponding quantum bit-flip event error can be described by an n-dimensional vector,

$$\rho_{i,j}(e_x)=(\underbrace{0,\ldots,0}_{i-1},e_x,\underbrace{0,\ldots,0}_{n-j}) \tag{3}$$

Clearly, there are at most $n-l(e_x)$ possible event errors which start at positions i corresponding to e_x.

Mathematically, the QBEE, i.e., e_x, can be obtained by the mapping $e_x=\varphi(\alpha)$ that transforms $\alpha\in\{0,1\}^n$ to e_x by remaindering the bits between the first and last bit “1”s in the vector α. Let e_{x_1} and e_{x_2} be two QBEEs with lengths of $l(e_{x_1})$ and $l(e_{x_2})$. Without loss of generality, suppose $l(e_{x_1})\le l(e_{x_2})$. The addition of two QBEEs can be defined as $e_{x_3}=e_{x_1}+e_{x_2}$ with the following rules: a) zero pads e_{x_1} at the beginning such that the new vector has length $l(e_{x_2})$. b) Add the resulting vector to e_{x_2} in $GF(2)$. Denote a set of distinct QBEEs by $E_x=\{e_x^{(1)},\ldots,e_x^{(M_x)}\}$ and lengths of the respective error events in E_x by $l(E_x)=\{l(e_x^{(1)}),\ldots,l(e_x^{(M_x)})\}$, where $M_x=|E_x|$ is the size of E_x. Then a QBEE set E_x is closed if and only if $\varphi(e_x^{(p)}+e_x^{(q)})\in E_x$ for $e_x^{(p)},e_x^{(q)}\in E_x$ with $p\neq q$.

In a similar way, one can describe the quantum phase error event (QPEE) with the length of $l(e_z)$ by $e_z=(b_1,b_2,...,b_{l(e_z)})$, with the corresponding quantum phase event error expressed by $\rho_{i,j}(e_z)=(\underbrace{0,...,0}_{i-1},e_z,\underbrace{0,...,0}_{n-j})$.

Based on e_x and e_z, the quantum flip-phase error event (QFEE) may be described by two binary vectors of the length of $l(e_y)$ as $e_y=(a_1,a_2,...,a_{l(e_y)}\,|\,b_1,b_2,...,b_{l(e_y)})=(e_x\,|\,e_z)$, where $a_1=b_1=a_{l(e_y)}=b_{l(e_y)}=1$ and $a_k=b_k\in\{0,1\}$ $(1\le k\le l(e_y))$.Define the corresponding error by

$$\rho_{i,j}(e_y)=(\underbrace{0,...,0}_{i-1},e_x,\underbrace{0,...,0}_{n-j}\,|\,\underbrace{0,...,0}_{i-1},e_z,\underbrace{0,...,0}_{n-j}) \tag{4}$$

In terms of Eq.(1), errors rising e_x, e_z and e_y in a quantum code may be represented by unitary operators X^{α}, X^{β}, $X^{\varepsilon_1}Z^{\varepsilon_2}$, respectively, where $\varepsilon_1,\varepsilon_2\in\{\alpha,\beta\}$. Suppose a basis $\{|v_i\rangle: v_i\in\{0,1\}^n, i=1,...,2^n\}$ in $H^{\otimes n}$, then one has $X^{\alpha}|v_i\rangle=|v_i+\alpha\rangle$, and $Z^{\beta}|v_i\rangle=(-1)^{v_i\cdot\beta}|v_i\rangle$.

The error operator of QBEE, QPEE and QFEE may be generalized by $p(E^{(m)})=(-1)^{\lambda}X^{\alpha}Z^{\beta}$ where $\lambda\in\{0,1\}$. Obviously, QBEE, QPEE and QFEE are especial cases of $E^{(m)}$ corresponding to $\beta=0,\alpha=0$, and $\alpha=\beta$ respectively. In addition, there is a more general event error with $\alpha\ne\beta$. Thus, the quantum error event may be generalized by the following vector,

$$e^{(m)}=(e_x^{(m)}\,|\,e_z^{(m)})=(\varphi(\alpha^{(m)})\,|\,\varphi(\beta^{(m)})), \tag{5}$$

where $1\le m\le M_x+M_z$, $\varphi(\alpha^{(m)})=(a_1^{(m)},a_2^{(m)},...,a_{l(e_x^{(m)})}^{(m)})$, $\varphi(\beta^{(m)})=(b_1^{(m)},b_2^{(m)},...,b_{l(e_z^{(m)})}^{(m)})$.

2 Construction of Quantum Event Error Correction Code

In this section, we show how to construct the stabilizer quantum code for event-error correction. Since a quantum error correction code can detect both quantum bit-flip event errors and quantum phase event errors at the same time and location (errors corresponding to QFEEs occur in this case), only QBEEs and QPEEs need to be considered in the following sections.

2.1 Syndrome of quantum event-error detection code

The particular code we wish to present may be best described by using the stabilizer formalism[14], which provides an elegant and simple way to understand the process of the encoding operations.

A stabilizer quantum code $((n,2^{n-k}))$ is defined to be a vector space V_s stabilized by a subgroup S of Pauli group G_n on n qubits, such that $S(-I\notin S)$ have k independent and commuting generators denoted by $\{g_i:i=1,2,\dots,k\}$. There is a good way of expressing these generators by exploiting the check matrix H which is a $K\times 2n$ matrix denoted by

$$H=\begin{bmatrix}\alpha_1\,|\,\beta_1\\ \vdots\,|\,\vdots\\ \alpha_k\,|\,\beta_k\end{bmatrix}, \tag{6}$$

where $\alpha_i,\beta_i\in\{0,1\}^n,(1\le i\le k)$, and the i-th row is the generator g_i described in the same way as in Eq.(5).

Consider a stabilizer $S=\{g_i:1\le i\le k\}$ with k generators, and the event error operator,

$$e_i=X^{\alpha_i}Z^{\beta_i}, 1\le i\le k. \tag{7}$$

Obviously, $e_i^2=I$ and $e_ie_j=e_je_i$, where $\alpha_i=0$ or $\beta_i=0$. Then a set of vectors $x\in H^n$, which satisfies $e_i|x\rangle=|x\rangle$, forms a $(n-k)$-dimensional stabilizer quantum code Q.

Suppose a quantum event-error correction code Q be described by a $(k_1+k_2)\times 2^n$ check matrix H with $k=k_1+k_2$. Especially, based on Eq.(6), the check matrix can be expressed as,

$$H=\begin{bmatrix}H_{k_1\times n}^{(x)} & 0\\ 0 & H_{k_2\times n}^{(z)}\end{bmatrix}=\begin{bmatrix}h_1^{(x)}...h_n^{(x)} & 0\\ 0 & h_1^{(z)}...h_n^{(z)}\end{bmatrix} \tag{8}$$

Theorem 2.1. *H* is a totally singular matrix if and only if

$$H^{(x)}_{k_1\times n}\cdot(H^{(z)}_{k_2\times n})^T=0_{k_1\times k_2} \tag{9}$$

Let *L* be the length of the longest quantum error event in *E*, i.e., $L=\max\{l(E_x),l(E_z)\}$. To correct any event error $\rho_{i,j}(e)$, the linear combination of the columns of *H* from *i* to *i+L–1*, from *n+i* to *n+i+L–1* and the event error $\rho_{i,j}(e)$ has to be nonzero. Especially, to correct an event error such that Eq.(5) for arbitrary *m* and $l(e^{(m)})$, the syndrome can be composed as,

$$s_i^m=\sum_{t=1}^{\min\{l(e^{(m)}),n-i\}}\left(\begin{pmatrix}h_{i+t}^{(x)}\\0\end{pmatrix}b_t^{(m)}+\begin{pmatrix}0\\h_{i+t}^{(z)}\end{pmatrix}a_t^{(m)}\right) \tag{10}$$

In matrix form, S_i^m can be rewritten as,

$$s_i^m=\sum_{t=1}^{\min\{l(e^{(m)}),n-i\}}\begin{pmatrix}h_{i+t}^{(x)}b_t^{(m)}\\h_{i+t}^{(z)}a_t^{(m)}\end{pmatrix} \tag{11}$$

where $l(e^{(m)})=\max\{l(e_x^{(m)}),l(e_z^{(m)})\}$. If $l(e_x^{(m)})\neq l(e_z^{(m)})$ (without loss of generality, suppose $l(e_x^{(m)})\leq l(e_z^{(m)})$, zero pads $e_x^{(m)}$ at the beginning such that the resulting vector has the same length as $l(e_z^{(m)})$ to compute s_i^m. To correct such an event error, the above syndrome must satisfy

$$s_i^m\neq 0 \tag{12}$$

The syndrome is a k_1+k_2 dimension vector, which shows the error event type *m* and the location *i* of the detected errors. First k_1 rows of vector s_i^m with nonzero show errors from the QPEEs corresponding to $H^{(x)}_{k_1\times n}$, and the last rows k_2 of vector with nonzero show errors from the QBEEs corresponding to $H^{(z)}_{k_2\times n}$. Furthermore, from the number of “1”s in the syndrome, one may obtain the exact number of quantum random errors occurring in the corresponding error event. Hence, the syndrome shows us not only the type of error event, but also the length, the location and even the total number of random errors occurring in the error event, which can be shown by,

$$\underbrace{...,\overbrace{i,i+1...,i+l(e_x^{(m)})}^{\det ected-errors},...}_{n} \tag{13}$$

In general, if the event error $\rho_{ij}(\tilde{e}_\tau)$ for $\tau=x,z$, which is detected by the syndrome s_i^m, is the unitary operator, the error in the quantum code can be corrected. To correct such an event error and recover the original state, one only needs to carry out the operator $\rho_{ij}(\tilde{e}_\tau)$ again on the received quantum states.

However, to correct errors corresponding to more error events, one has to design the compositional syndrome. Let $\Xi_\tau^{(m)}=(0_{(L-l(e_\tau^{(m)}))\times 1},e_\tau^{(m)})$, which is obtained by arranging $L-l(e_\tau^{(m)})$ zeros in front of $e_\tau^{(m)}$. According to Eq.(5), one obtains the error matrix

$$\Xi=\begin{bmatrix}\Xi^{(x)}_{M_x\times L} & 0\\ 0 & \Xi^{(z)}_{M_z\times L}\end{bmatrix}=\begin{bmatrix}\Xi_{11}^{(x)}...\Xi_{1L}^{(x)} & \Xi_{11}^{(z)}...\Xi_{1L}^{(z)}\\ \vdots & \vdots\\ \Xi_{M1}^{(x)}...\Xi_{ML}^{(x)} & \Xi_{M1}^{(z)}...\Xi_{ML}^{(z)}\end{bmatrix} \tag{14}$$

where $M=M_x+M_z$. Let $\psi_j^{(x)}=(h_{1j}^{(x)},h_{2j}^{(x)},...,h_{k_1,j}^{(x)})^T$ and $\psi_j^{(z)}=(h_{1j}^{(z)},h_{2j}^{(z)},...,h_{k_2,j}^{(z)})^T$ for 1≤*j*≤*L* be columns of matrix $H^{(x)}_{k_1\times n}$ and $H^{(z)}_{k_2\times n}$ respectively. Then vectors,

$$\xi_j^{(x)}=(h_{1j}^{(x)},h_{2j}^{(x)},...,h_{k_1j}^{(x)},\underbrace{0,...,0}_{k_2})^T,\ \xi_j^{(z)}=(h_{1j}^{(z)},h_{2j}^{(z)},...,h_{k_2j}^{(z)},\underbrace{0,...,0}_{k_1})^T \tag{15}$$

denote columns of the check matrix H. A syndrome S for general quantum event errors from the error matrix Ξ may be composed as

$$S=\begin{bmatrix}\sum_{h=1}^{L}\begin{bmatrix}\psi_{(0+h)}^{(x)}\Xi_{1h}^{(z)}\\ \psi_{(0+h)}^{(z)}\Xi_{1h}^{(x)}\end{bmatrix} & \sum_{h=1}^{L-1}\begin{bmatrix}\psi_{(1+h)}^{(x)}\Xi_{1h}^{(z)}\\ \psi_{(1+h)}^{(z)}\Xi_{1h}^{(x)}\end{bmatrix} & \cdots & \sum_{h=1}^{1}\begin{bmatrix}\psi_{(L-1+h)}^{(x)}\Xi_{1h}^{(z)}\\ \psi_{(0+h)}^{(z)}\Xi_{1h}^{(x)}\end{bmatrix}\\ \vdots & \vdots & \vdots & \vdots\\ \sum_{h=1}^{L}\begin{bmatrix}\psi_{(0+h)}^{(x)}\Xi_{Mh}^{(z)}\\ \psi_{(0+h)}^{(z)}\Xi_{Mh}^{(x)}\end{bmatrix} & \sum_{h=1}^{L-1}\begin{bmatrix}\psi_{(1+h)}^{(x)}\Xi_{Mh}^{(z)}\\ \psi_{(1+h)}^{(z)}\Xi_{Mh}^{(x)}\end{bmatrix} & & \sum_{h=1}^{1}\begin{bmatrix}\psi_{(L-1+h)}^{(x)}\Xi_{Mh}^{(z)}\\ \psi_{(0+h)}^{(z)}\Xi_{Mh}^{(x)}\end{bmatrix}\end{bmatrix} \tag{16}$$

The syndrome S is an MKL matrix. Denote rows of S from $m(k-1)+1$ to mk by S^m, which consists of the syndrome of event error type m calculated by using check matrix H. By using the similar trick to analyze the syndrome $s_i^{(m)}$ of an error event, one can detect lengths, numbers, and locations of the occurring random errors corresponding to multiple error events.

For convenience, we define two "existence" functions $\varepsilon_\Xi(\cdot),\varpi_\Xi(\cdot)$. If $\varepsilon_\Xi(S^H)=1$, Then S^H must be nonzero matrix for the given matrices Ξ and H, otherwise, $\varepsilon_\Xi(S^H)=0$. In a similar way, if $\varpi_\Xi(H)=1$, H must be totally singular, otherwise, $\varpi_\Xi(H)=0$.

2.2 An algorithm for construction of quantum event-error correction codes

In this subsection, we investigate how to construct a quantum event-error correction code by exploiting the syndrome S. Obviously, to construct such a code, the check matrix which gives rise to a nonzero syndrome $s_i^{(m)}$ for arbitrary error event $e^{(m)}$ needs to be yielded explicitly. In the following we present an approach for constructing such a check matrix.

For a given error matrix Ξ, firstly, we choose vectors $\psi_1^{(x)},\psi_2^{(x)},\dots,\psi_L^{(x)}$ of k_1-dimension and vectors $\psi_1^{(z)},\psi_2^{(z)},\dots,\psi_L^{(z)}$ of k_2-dimension as L columns of $H_{k_1\times n}^{(x)}$ and $H_{k_2\times n}^{(z)}$ of the check matrix in Eq.(8) respectively, to create an initial matrix

$$\psi_0=\begin{bmatrix}\psi_1^{(x)},\psi_2^{(x)},\dots,\psi_L^{(x)} & 0\\ 0 & \psi_1^{(z)},\psi_2^{(z)},\dots,\psi_L^{(z)}\end{bmatrix} \tag{17}$$

such that $\varepsilon_\Xi(S_0{}^{\psi_0})=1$ and $\varpi_\Xi(\psi_0)=1$. Then create the second matrix ψ_1 by cutting the first column and adding a new column $h_1=\{h_1^{(x)},h_1^{(z)}\}$ in the matrix ψ_0, i.e.,

$$\psi_1=\begin{bmatrix}\psi_2^{(x)},\psi_3^{(x)},\dots,h_1^{(x)} & 0\\ 0 & \psi_2^{(z)},\psi_3^{(z)},\dots,h_1^{(z)}\end{bmatrix} \tag{18}$$

Finally, matrices $\psi_1,\psi_2,\dots,\psi_{n-L}$ are created in a similar way by adding $h_1,h_2,\dots,h_{n-L}$ and cutting $\psi_1^{(x)},\psi_2^{(x)},\dots,\psi_{n-L}^{(x)}$ respectively until the $n-L$-th matrix. Each matrix ψ_k satisfies the following conditions,

$$\varepsilon_\Xi(S_k{}^{\psi_k})=1,\ \varpi_\Xi(\psi_k)=1 \tag{19}$$

where $S_k{}^{\psi_k}$ denotes a syndrome of ψ_k for $k=0,1,\dots,n-L$. Utilizing the initial matrix ψ_0 and added columns $h_1,h_2,\dots,h_{n-L}$, the matrix $\tilde{H}$ is obtained,

$$\tilde{H}=\begin{bmatrix}\psi_1^{(x)},\psi_2^{(x)},\dots,\psi_L^{(x)},h^{(x)} & 0\\ 0 & \psi_1^{(z)},\psi_2^{(z)},\dots,\psi_L^{(z)},h^{(z)}\end{bmatrix} \tag{20}$$

where $h^{(\tau)}=(h_1^{(\tau)},h_2^{(\tau)},\dots,h_{n-L}^{(\tau)})$.

The above procedure may be done by a finite quantum condition machine (FQCM). Define now a FQCM whose starting condition is labeled by an arbitrary matrix ψ_0 such that $\varepsilon_\Xi(S_0{}^{\psi_0})=1$ and $\varpi_\Xi(\psi_0)=1$, and the transition (edge) condition is labeled by h_k from each existing matrix ψ_0 to any existing matrix ψ_k. Since all matrices $\psi_k(k=0,1,\dots,n-L)$ satisfy Eq.(19), the check matrix $\tilde{H}$ may be obtained by walking through the FQCM and reading off the edge label h_k in turns.

Many paths may be chosen in the FQCM for completing the check matrix, i.e., many h_k s can be chosen at each step for creating ψ_k. Since paths that produce a large number of different syndromes do not always result in good codes because of higher probability of producing zero elements of the vector $s_j^{(m)}$ in syndrome S, how to choose an optimal h_k becomes an important problem. Consider a path of length n, and denote the set of positions of all event-errors by $Q_s^{(m)}=\{l:1\le l\le n, s_l^{(m)}=s\}$, where error event of type m gives rise to the same syndrome s. Let λ_s^m be the size of Q_s^m (i.e., $\lambda_s^m=|Q_s^m|$), the occurring probability of two quantum error events that produce zero element of the syndrome (ignoring the edge effect) is

$$P_{miss,2}=\sum_{s_1,s_2}\sum_{m_1,m_2}P_{m_1}P_{m_2}\frac{\lambda_{s_1}^{m_1}\lambda_{s_2}^{m_2}}{n\cdot n} \tag{21}$$

In a similar way, the occurring probability of three or more quantum error events can be obtained. Then the total probability of producing a zero s of syndrome S is

$$P_{miss}=\sum_{m>1}P_{miss,m} \tag{22}$$

The condition for the optimal h_k is to minimize p_{miss}.

Based on the above theory, an algorithm comes for the construction of quantum event-error correction code with minimizing the probability of producing zero element of s in syndrome S.

An algorithm for finding the check matrix H:

Step.1 For a given error Ξ, find all ψ_0 and ensure $\varepsilon_{\Xi}(S_0{}^{\psi_0})=1$ and $\varpi_{\Xi}(\psi_0)=1$. Denote the set of all possible ψ_0 by V.

Step.2 Select randomly a matrix ψ_0 from V.

Step.3 Let $H_0=\psi_0$.

Step.4 For $1\le k\le n-L$ repeat:

- For all edges h_k emanating from ψ_k do:

(1) Create $\Omega_{k_1}(h_{k_1})=\psi_{k_1}$, such that Eq.(19). (2) Assuming that Ω_{k_1} is a check matrix, calculate $P_{miss}(\Omega_{k_1})$. (3) Find a label h_k which minimizes $P_{miss}(\Omega_{k_1})$. (4) Continue (5). Let $\psi_k=\psi_{k_1}$.

- continue

Step.5 $H=\tilde{H}$ such that Eq.(20)

It is necessary to remark that the $2n$-dimension vector $(u_x\,|\,u_z)$ satisfying $H_1\cdot u_z{}^T+H_2\cdot u_x{}^T=0$ forms the quantum event-error detection code Q. These vectors are generated by $G=(G_x\,|\,G_z)$, where $H_1\cdot G_z{}^T+H_2\cdot G_x{}^T=0$ for $H_1=(H^x,0)^T$, and $H_2=(0,H^z)^T$.

3 Discussion and Conclusion

A quantum event-error correction scheme is put forward for correcting any event errors from the listed error event sets. And, the process of the construction is introduced based on the syndrome inferred in this paper. Since the designed quantum code can easily point out the exact location of single errors symbol by utilizing the syndrome, it may be more convenient to use such a code to correct these different errors in the complex quantum channels. In fact, it is easy to prove that the code can detect random quantum errors with an upper bound $3(n-l(e_\tau))\sum_{0\le\lambda\le l(e_\tau)}\binom{n+\lambda-2}{\lambda-1}|E_\tau|^\lambda$, where $l(e_\tau)$ is the length of the error event and $|E_\tau|$ is the size of the listed error event. For the other hand, the code can detect quantum burst errors with length d. Therefore, compared with the two kinds of previous QECCs (i.e., quantum random error-correction code and quantum burst error-correction code), the proposed code has an advantage of detecting two different kinds of errors at the same time, namely, the code can detect a fraction of errors which consists of either quantum random errors or quantum burst errors, and it hence has more ability to detect quantum errors with more efficiency.

References:

[1] Zeng GH, Keitel CH. Inhibiting decoherence via ancillary processes. Physical Review A, 2002,66(2):022306.

[2] Ashikhmin AE, Barg AM, Knill E, Litsyn SN. Quantum error detection I: Statement of the problem. IEEE Trans. on Information Theory, 2000,46(3):778−787.

[3] Calderbank AR, Shor PW. Good quantum error-correcting codes exist. Physical Review A, 1996,54(2):1098−1105.

[4] Knill E, Laflamme R. A theory of quantum error correcting codes. Physical Review A, 1997,55(2):900−911.

[5] Feng K, Ma Z. A finite gilbert–varshamov bound for pure stabilizer quantum codes. IEEE Trans. on Information Theory, 2004, 50(12):3323.

[6] Shor PW. Scheme for reducing decoherence in quantum computer memory. Physical Review A, 1995,52(4):2493−2496.

[7] Steane AM. Multiple particle interference and quantum error correction. Proc. of the Royal Society A: Mathematical, Physical and Engineering Sciences, 1996,452(1954):2551−2577.

[8] Calderbank AR, Rains EM, Shor PW, Sloane NJA. Quantum error correction and orthogonal geometry. Physical Review Letters, 1997,78(3):405−408.

[9] Gottesman D. A class of quantum error-correcting codes saturating the quantum hamming bound. Physical Review A, 1996,54(3): 1862−1868.

[10] Chen H. Some good quantum error-correcting codes from algebraic–geometric codes. IEEE Trans. on Information Theory, 2001, 47(5):2059.

[11] Grassl M, Klappenecker A, Roteler M. Graphs, quadratic forms, and quantum codes. In: Proc. of the IEEE Int’l Symp. Information Theory. Lausanne, 2002. 45.

[12] Kawabatax S. Quantum interleaver: Quantum error correction for burst error. Journal of the Physical Society of Japan, 2000,69(11): 3540−3543.

[13] Vatan F, Roychowdhury VP, Anantram MP. Spatially correlated qubit errors and burst-correcting quantum codes. IEEE Trans. on Information Theory, 1999,45(5):1703.

[14] Nielsen MA. Quantum Computation and Quantum Information. Cambrige: Cambrige University Press, 2003. 454−674.

GUO Ying was born in 1975. He is a Ph.D. candidate at the Department of Electronic Engineering, Shanghai Jiaotong University. His current research areas are quantum information and quantum compution.

MA Shao-Jun was born in 1958. He is a professor at the Science of School, Laiyang Agriculture College, Shandong province. His current research areas are system theory and quantum compution.

ZENG Gui-Hua was born in 1965. He is a professor at the Department of Electronic Engineering, Shanghai Jiaotong University. His current research areas are quantum information and quantum compution.