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A Novel Method for Corner Detection Based on Conditional Skeleton*

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Abstract: A source image is regarded as a polygon, and the extended line of the morphologic conditional skeleton must cross the vertex of a polygon. The convex corners can be obtained through detecting the zero radius of the maximum plate on morphologic skeleton. All the results of corner detection are given by logical hetero-or operation between two corner sets of source image and its supplemental image. In addition to the pseudo lattice, corners which are generated by discrete sampling can also be eliminated. This method has very high value in exactly locating the corner, can reduce the computational cost, only requires simple hardware equipment, and is convenient for parallel processing. Because of using large plate to detect corner, this method is good for noise filtering and can be popularized to gray images by using two kinds of large plate.

Key words: corner detection; conditional skeleton; morphology; supplemental image; minimum plate

A corner is defined as the junction point of two or more straight-line edges. Corners are special features in an image. They are very useful in computing optical flow, pattern recognition, image matching, motion analysis and so on. The earliest corner detection method involved segmenting an image into regions first and then using a chain code to represent the object boundaries. The work done by Rosenfeld[1] to detect a corner using *k*-curvature is the motivation for this work, but it is too simple to solve the problems completely. Asada[2] used Gaussian smoothness technique and cubic *B*-splines to calculate *k*-curvature, detect the discontinuous points on a curve, and then find the corners. Other developments based on Asada's work include those by Langride[3], Gerard Medioni[4] and Fischler[5]. Another method for gray image corner detector is proposed by Krishnan *et al.*[6], in which they described a corner as the production of the sine in x and an exponential in its direction in a portion of the mask and the production of two sines in x and y directions in the remaining portion. However, these methods need complex computation and are difficult to implement using hardware.

In this paper, a novel method for corner detection based on mathematical morphologic skeleton is proposed.

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The source image is regarded as a polygon, and the corners can be obtained through detecting the zero radius of the maximum plate on morphologic skeleton. In particular, the corners of a concave vertex can be obtained using the supplemental image. All the results of corner detection are given by logical hetero-or operation between two corner sets of the source image and its supplemental image. In addition, the pseudo lattice corners where are generated by discrete sampling can be eliminated. Experiments show that this method works well, performs at a high computational speed and is easy to implement using hardware.

1 The Method of Skeleton Extracted from Continuous Sets

Definition 1. Suppose the set X and structure element B are two arbitrary sets of points in E^2 space. Minkowski addition and subtraction can be defined as follows:

$$X \oplus B = \{X : \tilde{B}_l \cap X \neq \varnothing\}$$

$$X \Theta B = \{x : \tilde{B}_r \subset X\}$$

where, $\tilde{B}_l = \{x - b : b \in B\}$, $\tilde{B}_r = \{x + b : b \in B\}$.

Definition 2. Suppose $X \in E^2$, B is a closed unit sphere in E^2. If rB_x ($r \geqslant 0$ is a real number) satisfies the following three conditions:

1) $rB_{\subset} X$

2) There is not any other structure element $r'B_{x'} \subset X$, $r'B_{x'} \supset rB_x$.

3) rB_x is the maximum plate in set X, point x is the center and r is the radius. Point x is called the skeleton point in set X. All skeleton points are called the skeleton of set X and denoted as $SK(X)$, and the center of the maximum plate whose radius is r is denoted as $SK_r(X)$, all of which appear $SK(X) = \bigcup_{r>0} SK_r(X)$.

If $D(x)$ is the maximum plate on skeleton point x, then set X can be reconstructed completely using skeleton, thus, $X = \bigcup_{x \in SK(X)} D(x)$.

Definition 3. Suppose $X \in E^2$, B is a convex structure element (including coordinate origin), and for any $x \in E^2$, $r_{X|b}(x) = \begin{cases} \mathrm{Sup}\{r : rB_x \subset X\} & x \in X \\ 0 & x \notin X \end{cases}$ is called the constructing characteristic function of set X about structure element B. $r_{X|b}(x)B_x$ denotes the maximum constructing characteristic element corresponding to X. When for B is a closed unit sphere, the constructing function is denoted as $r_X(x)$.

Definition 4. Let $X \in E^2$, B is a closed unit sphere, then $SK^{\alpha}(X) \stackrel{\Delta}{=} \left\{x : \lim_{t \to 0} \frac{d(x, X\Theta(r_X(x)+t)B)}{t} > \alpha\right\}, \alpha \geqslant 1$ is called the condition skeleton of set X, and $\lim_{t \to 0} \frac{d(x, X\Theta(r_X(x)+t)B)}{t}$ is called the contract ratio of skeleton point x, where d denotes the Euclid distance.

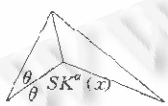


Fig. 1 Conditional skeleton halve the internal angle

If $x \in SK(X)$, and $d(x, X\Theta(r_X(x)+t)B) - t$, so $SK^{\alpha}(X) \subset SK(X)$, thus the set of conditional skeleton is a proper subset of skeleton.

Suppose X is a polygon and one of its internal angles is 2θ. When $\alpha \leqslant 1/\sin\theta$, $SK^{\alpha}(X)$ is the bisector of this internal angle; otherwise, when $\alpha \leqslant 1/\sin\theta$, it is a null set. To an appointed α, $SK^{\alpha}(X)$ filters those bisectors cross the internal angle 2θ which satisfy the condition ($\sin\theta > 1/\alpha$), it is shown in Fig. 1. Formula (1) gives a new method to compute condition skeleton.

$$SK_r^{\alpha}(X) = (X\Theta rB) - (X\Theta rB)_{drB} \oplus (\alpha - 1)drB \tag{1}$$

where structure element B is a unit plate, dr is a positive minuteness real number, drB is a minuteness plate with a radius dr, and $(\cdot)_{drB}$ is a symbol of morphological opening for drB to $(\cdot)$; thus

$$(X\Theta rB)_{drB} = [(X\Theta rB)\Theta drB] \oplus drB$$

For all appearances, when $\alpha=1$,

$$SK^1(X)=(X\Theta rB)-(X\Theta rB)_{drB}$$

$$SK^1(X)=SK(X)$$

The skeleton of X is the conditional skeleton of X when $\alpha=1$. According to the characters of the conditional skeleton, the extended line of the conditional skeleton must cross the vertex of a polygon. So conditional skeleton halves the internal angle of the polygon X, and for each vertex of X there is one extended line of the conditional skeleton crossing it. The corner of X must be on extended line of the conditional skeleton, and the maximum plate of the skeleton point on the conditional skeleton tends to zero when near to the corner.

Then the corner is denoted as $\qquad SK_0^1(X)=X \quad X_{drB}=X-(X\Theta drB)\oplus drB$ (2)

Corner detection is the simplest method in this case, as shown in Fig. 2.

Let $X=Q_1PQ_2$ be a polygon, so:

$(X\Theta drB)\oplus drB=Q_1\,\widehat{AB}Q_2,\ X-(X\Theta drB)\oplus drB=PA\,\widehat{AB}PB$

The result of corner detection is the region made up of PA, $\widehat{AB}$, PB. The greater drB is, the larger are the regions that can be detected. So, we can use large dr plate to detect corner. Given a threshold T, when $dr>T$, all corners with radius larger than dr can be detected, and the noise with radius less than dr is filtered. This result of corners detection by large plate is regarded as the first step for corner detection, then the accurate corner detection is conducted by using operator $SK_0(X)$.

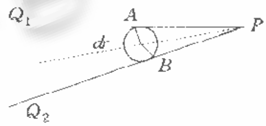


Fig. 2 The result of corner detection, with the dash denoting the skeleton

Because $SK_0(X)=SK_0^1(X)$, the internal angle $\theta(\sin\theta<1/\alpha=1)$ can only be detected according to the character of the condition skeleton. The internal angle should be less than 180°, thus operator $SK_0(X)$ is only for detection of the convex corner rather than the concave corner. In this case, we should calculate the supplemental set X^C of X first, then calculate the corner of X^C. All the corners of X and X^C comprise the whole corner set of X.

2 The Method of Corner Detection for Gray Image

Corners are identified where the direction changes rapidly in a chain code in discrete binary image, but drB cannot be set too large in discrete image and should be expressed as an accurate plate. As a result, Formula (2) should be changed to a generalized morphologic opening pursuant to the following Formula (3):

$$SK_0(X)=X-(X\Theta B_1)\oplus B_2 \tag{3}$$

The radius of B_2 is one pixel larger than the radius of B_1 in discrete binary image, and X can be reconstructed completely. Then, the two kinds of minimum plate S_4 and S_8 are defined, and shown in Fig. 3.

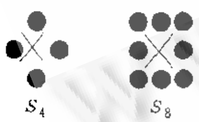


Fig. 3 Two kinds of discrete minimum plate

Suppose the binary image is I, then

$$I\oplus S_4(i,j)=\max\{I(i+1,j),I(i,j+1),I(i,j),I(i-1,j),I(i,j-1)\}$$

$$I\Theta S_4(i,j)=\min\{I(i+1,j),I(i,j+1),I(i,j),I(i-1,j),I(i,j-1)\}$$

If the radiuses of B_2 and B_1 are unequal, one needs to use a more complex calculation. The following Formula (4) is used as a corner detection operator,

$$Cor_1=I-(I\Theta S_8)\oplus S_8 \tag{4}$$

This operator can only detect the convex corner whose internal angle is less than 90° but miss those corners whose internal angle is less than 180° and larger than 90°. It can also remove the pseudo lattice corner generated by discrete sampling, which means that this operator suits to detection of the acute angle corner.

For detection of an obtuse angle corner, B_2 should be a proper subset of B_1 and is much similar to a plate in Formula (3). As a result, the following Formula (5) is used as a new corner detection operator,

$$Cor_2=I-(I\Theta S_8)\oplus S_4 \tag{5}$$

This operator is not only used for acute angle corners but also for obtuse angle corners, such that all the points whose directions change rapidly in chain code will be detected. Importantly, the diagonal at 45° direction will be detected as a continuous line because there is a little sidestep in each adjacent point on the diagonal. The supplemental image I^C of source image I is calculated for detecting concave corner, but the boundaries of I and I^C are not the same and have one pixel discrepancy. After executing a dilation calculation for I^C, $I'=I^C\oplus S_4$, I' and I have the same boundary. The Cor_3 operator in the following Formula (6) is used for I' corner detection.

$$Cor_3 = I' - (I'\Theta S_8)\oplus S_4 \tag{6}$$

The concave corner of I is detected using this operator, and the diagonal at 45° direction is detected as a continuous line which is the same as that obtained using Cor_2. As a result, all the corners are denoted as the following Formula (7):

$$Cor = Cor_2 A Cor_3 \tag{7}$$

where A denotes the hetero-or operation of two images. Figure 4 shows the flow chart of the corner detection method based on morphologic skeleton in binary image.

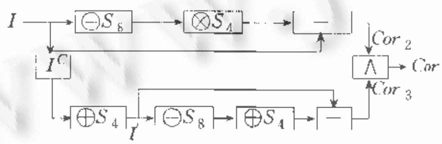


Fig. 4 Flow chart of the corner detection method based on morphologic skeleton in binary image

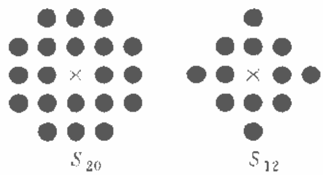


Fig. 5 Two kinds of discrete minimum plate for gray image

This method can be used to gray images. In gray image, the value of pixel gray degree can be regarded as the high value in 3D coordinate system, and the polygon of binary image becomes a polyhedron. We can use the conditional skeleton to detect corners. Because the edges in gray image are smooth sidesteps, these edges can be detected as corners. According to Formula (1), to locate accurate corner, the minuteness radius dr of plate is needed, but for smooth sidestep edges, the radius dr of plate should be large. A number of experiments show that two kinds of minimum plate S_{20} and S_{12} are good for gray image corner detection (Fig. 5).

$$Cor_4 = I - (I\Theta S_{20})\oplus S_{12}$$

$$Cor_5 = I' - (I'\Theta S_{20})\oplus S_{12}$$

S_{20} can be decomposed into two structure elements S_8, S_4 Minkowski addition and S_{12} can be decomposed into two structure elements S_4, S_4 Minkowski addition, so

$$S_{20} = S_{12}\oplus S_4$$

$$S_{12} = S_4\oplus S_4$$

$$Cor_6 = I - (I\Theta S_8\Theta S_4)\oplus S_4\oplus S_4 \tag{8}$$

$$Cor_7 = I' - (I'\Theta S_8\Theta S_4)\oplus S_4\oplus S_4 \tag{9}$$

$$Cor_g = Cor_6 A Cor_7 \tag{10}$$

For gray image

$$I\oplus S_3(i,j) = \max\{I(i+1,j+1), I(i+1,j), I(i+1,j-1), I(i,j+1), I(i,j), I(i,j-1), I(i-1,j+1), I(i-1,j), I(i-1,j-1)\}+v$$

$$I\Theta S_8(i,j) = \min\{I(i+1,j+1), I(i+1,j), I(i+1,j-1), I(i,j+1), I(i,j), I(i,j-1), I(i-1,j+1), I(i-1,j), I(i-1,j-1)\}-v$$

v is a constant, it is related to image gray.

We use Cor_g operator for corner detection in gray image. Set a threshold V_T. If the value of pixel in Cor_g is greater than the threshold V_T, this point is regarded as a corner. This method is influenced by noise and the gray degree changes. It can detect points other than corners, thus, better method for gray image corner detection should be paid more attention to in the future research. Figure 6 shows the flow chart of the corner detection method for gray image.

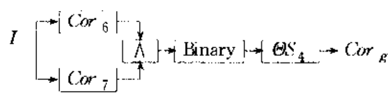
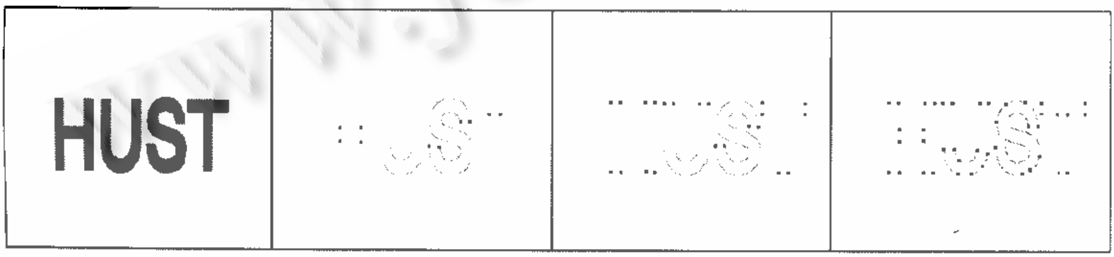


Fig. 6 Flow chart of the corner detection method based on morphologic skeletion in gray image

3 The Result of Experiment and Conclusion

Figure 7 shows the result of corner detection for English letters "HUST". (a) is the source image, and (b) is the result of corner detection using operator Cor_3. It shows that only the concave corners are detected. (c) is the result of corner detection using operator Cor_2 and shows that only the convex corners are detected. (d) is the result of hetero-or operation between (b) and (c), showing that the real corners in the source image are boosted up, and the pseudo lattice corners on the diagonal are eliminated.



(a) (b) (c) (d)

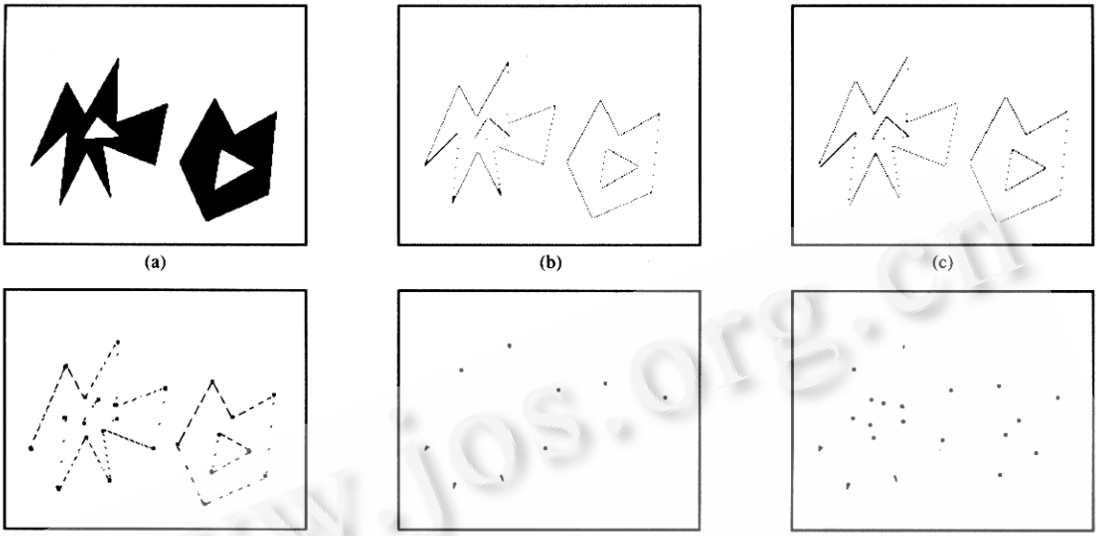
Fig. 7 The result of corner detection for English letters "HUST"

The result of corner detection of concave polygons with holes is shown in Fig. 8. (a) is the source image; (b) is the result of corner detection using operator Cor_2; (c) is the result of corner detection using operator Cor_3; (d) is the result of hetero-or operation between (b) and (c); (e) is the result of corner detection using operator Cor_1. Only the acute angle corners are detected. (f) is the result when the supplemental source image is operated by operator Cor_1. After (f) is hetero-or operated with (e), all real corners are detected, except for the obtuse angle corners.

There are $(1+4+8)N^2$ comparison calculations for an $N\times N$ pixels image by operator Cor_2, $(1-4+1+4+8)N^2$ calculations by operator Cor_3, and N^2 calculations by operator Cor. So there are only $32N^2$ comparison calculations and no any other complex calculations for Fig. 4. There are only $48N^2$ comparison calculations for Fig. 6 and can get the result of corner detection for gray image.

The experiments show that this method is very useful in locating the corners, can reduce the computational cost. It only needs simple hardware, is convenient for parallel processing, is good for noise filtering and is anti disturbance because of executing Minkowski subtraction first. The greater are the radiuses of structure elements B_1 and B_2, the larger are the regions for corner detection and the easier is corner estimation. Thus, the precision for location of a corner is reduced, and the computational cost is increased. The future research will focus on corner detection in gray images.

Figure 9 shows the result of corner detection for gray image by using operator Cor_g. (a) is the source gray image with noise, (b) is the result of corner detection, processed to binary value. There are many pseudo corners in (b).



(a) (b) (c) (d) (e) (f)

Fig. 8 The result of corner detection for concave polygons with holes

(a) (b)

Fig. 9 The result of corner detection for gray image by using operator Cor_g

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一种基于条件骨架的拐点检测方法

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摘要:把原始图像看成一个多边形,则形态条件骨架的延长线一定通过多边形的顶点,通过检测条件骨架中最大圆盘半径为零的点来获得凸点的拐点,并由它的补图来获得凹点的拐点.全部拐点由原图及补图的拐点进行逻辑异或操作获得并可去掉离散化后产生的网格拐点.此方法定位精度高、速度快,易于硬件实现和并行处理.由于先用较大圆盘做初步检测,可以降低噪声干扰.利用两种新的大圆盘,此方法可推广到灰度图像的拐点检测.

关键词:拐点检测;条件骨架;形态学;补图;极小圆盘

中图法分类号:TP391 **文献标识码**:A