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On Liveness and Safeness of Asymmetric Choice Nets*

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Abstract Liveness and safeness are important behavioral properties of nets (systems). Many powerful results have been derived for some subclasses of Place/Transition nets (systems). The aim of this contribution is to draw a general perspective of the liveness and safeness for Asymmetric Choice nets (AC nets). Firstly, this paper presents a sufficient and necessary condition for those AC nets which have liveness monotonicity and a polynomial time algorithm to decide if a given AC system is live and safe, and it satisfies liveness monotonicity. And then the sufficient and necessary conditions of (structural) liveness and (structural) safeness for two subclasses of AC nets (Strong I AC nets, Strong II AC nets) which have liveness monotonicity are presented.

Key words Asymmetric choice net, live, safe, structural liveness, structural safeness, liveness monotonicity, strong I AC net, strong II AC net.

1 Introduction

Liveness and safeness are main behavioral properties of Place/Transition (P/T) nets[1]. Liveness corresponds to the absence of global or local deadlock situations and safeness corresponds to the absence of overflow. For general P/T systems, it is difficult to analyze the liveness based on reachability graph. Thus, people concentrate on liveness of many useful subclasses of P/T nets. They hope to find ideal algorithms under some limitation.

At present, liveness and safeness analysis is easy for State Machine (SM), Marked Graph (MG)[2] and (Extended) Free Choice nets ((E)FC nets)[3~6], but, as we know, these mentioned subclasses of Petri nets are very simple. Many real-world systems are more complex. Although Asymmetric Choice nets (AC nets) are more general subclass of Petri nets, they still lack methods of analysis up to now. On the other hand, we know that liveness of safe (E)FC nets (SM or MG) can be decided in polynomial-time. We want to find a maximal subclass of safe Petri nets, whose liveness can be decided in polynomial-time. This paper gives some new results in this direction.

We know that general AC nets do not have liveness monotonicity[7,8]. This paper presents a sufficient and necessary condition for those AC nets which have liveness monotonicity and a polynomial-time algorithm to decide if a given AC system is live, safe and satisfy liveness monotonicity. Then the sufficient and necessary conditions of (structural) liveness and (structural) safeness for two subclasses of AC nets (Strong I AC nets, Strong

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II AC nets) which have liveness monotonicity are presented.

The paper is organized as follows. Section 2 gives the basic concepts and notations. Section 3 presents AC net theory. In Section 4, Strong I AC net theory is given. Section 5 presents Strong II AC net theory. Section 6 concludes the whole contribution

2 Basic Concepts and Notations

We assume the reader is familiar with the structure, firing rule, basic properties of net models[9] and elementary graph theory. However, in this section we recall some basic concepts and notations that are going to be used.

Definition 2.1.

1. A (Petri) net is a triple $N=(P,T;F)$ where
   (1) $P=\{p_1,p_2,\ldots,p_m\}$ is a finite set of places,
   (2) $T=\{t_1,t_2,\ldots,t_n\}$ is a finite set of transitions,
   (3) $F\subseteq(P\times T)\cup(T\times P)$ is a set of arcs (flow relation),
   (4) $P\cap T=\varnothing$ and $P\cup T\neq\varnothing$,
   (5) $dom(F)\cup cod(F)=P\cup T$ ($dom(F)=\{x\mid\exists\ y:(x,y)\in F\}$, $cod(F)=\{x\mid\exists\ y:(y,x)\in F\}$).
2. A Petri net $N=(P,T;F)$ is called Asymmetric Choice net iff

$$\forall\ (p,q)\in P\times P,\ p^{\bullet}\cap q^{\bullet}\neq\varnothing\Rightarrow p^{\bullet}\subseteq q^{\bullet}\text{ or }q^{\bullet}\subseteq p^{\bullet}.$$

3. A pair of a place p and a transition t is called a self-loop iff $(p,t)\in F\wedge(t,p)\in F$. A net is said to be pure iff it has no self-loops.

In the following, we only consider pure nets.

Definition 2.2. Let $N=(P,T;F)$ be a net and $\Sigma=(N,M)$ be a net system with the marking M.

1. The incidence matrix A of N is an $m\times n$ matrix of integers and its entry is given by

$$a_{i,j}=\begin{cases}1 & \text{if}\quad p_i\in t_j^{\bullet}\\ -1 & \text{if}\quad p_i\in{}^{\bullet}t_j\\ 0 & \text{otherwise}\end{cases}$$

2. A net N is P-connected iff $\forall\ x,y\in P$, there exists a directed path from x to y.
   A net N is strong connected iff $\forall\ x,y\in P\cup T$, there exists a directed path from x to y.
3. $t\in T$ is enabled at M iff $\forall\ p\in{}^{\bullet}t:M(p)\geqslant1$.
4. A transition t enabled at M can fire and after firing the marking is changed from M to M':

$$M'(p)=\begin{cases}M(p)+1 & \text{if}\quad p\in t^{\bullet}\\ M(p)-1 & \text{if}\quad p\in{}^{\bullet}t\\ M(p) & \text{otherwise}\end{cases}$$

   This is denoted by $M[t>M'$. For $M[t_1>M_1\ldots[t_i>M_i$, $i\geqslant1$, $M_1,\ldots,M_i$ are called reachable from M. The set of reachable markings from the marking M_0 is denoted by $R(M_0)$ or $R(N,M_0)$.
5. $t\in T$ is live for the system (N,M_0) iff $\forall\ M\in R(N,M_0)$, $\exists\ M'\in R(N,M)$: t is enabled at M'.
6. (N,M_0) is safe iff $\exists\ k\in Z$, $\forall\ p\in P$, $M\in R(N,M_0)$: $M(p)\leqslant k$. (Safe is the recommended term in net community, sometimes it is called bounded).
   N is structurally safe iff $\forall\ M_0$, and (N,M_0) is safe.
7. (N,M_0) is live iff $\forall\ t\in T$, and t is live.
   N is structurally live iff $\exists\ M_0$, and (N,M_0) is live.
8. $H\subseteq P$ is a siphon of N iff $H\neq\varnothing$ and ${}^{\bullet}H\subseteq H^{\bullet}$. A siphon is minimal iff it does not contain a siphon as a

proper subset.

$R\subseteq P$ is a trap iff $R\neq\varnothing$ and $R^{\bullet}\subset{}^{\bullet}R$. A trap is minimal iff it does not contain a trap as a proper subset.

9. An S-component N_1 of a net N is defined as a subnet generated by places in N_1 having the following two properties:

1) each transition in N_1 has at most one incoming arc and at most one outgoing arc;

2) N_1 is the net consisting of these places, all of their input and output transitions and their connecting arcs.

A Petri net $N=(P,T;F)$ is said to be covered by S-component iff $\forall p\in P$, $\exists$ S-component satisfy: $p\in$ S component.

3 AC Nets

3.1 Liveness monotonicity[10]

To prove liveness monotonicity theorem of AC net, first we give a lemma.

Lemma 3.1. Let $N=(P,T;F)$ be an AC net. If $H\subseteq P$ is a minimal siphon of N, then H is P-connected.

Proof. We first define a relation r in H:

$$\forall p_1,p_2\in H:\ p_1 r p_2 \Leftrightarrow \text{there exist directed paths from } p_1 \text{ to } p_2 \text{ and from } p_2 \text{ to } p_1.$$

Obviously, r is an equivalent relation in H.

Let $H/r=\{[p_1]_r,[p_2]_r,\ldots,[p_k]_r\}$ ($[p_i]_r=\{p_j\mid p_j\in H\wedge p_i r p_j\}$), where $[p_i]_r$ $(i=1,\ldots,k)$ is p_i equivalent class about r. To prove this lemma, we must prove: $k=1$, and reduction to absurdity is used here.

Assume $k\geqslant 2$, the directed graph G can be defined as follows: the node set of G is H/r, arc set of G is $\{([p_i]_r,[p_j]_r)\mid$ there exists directed path from p_i to p_j and $[p_i]_r\neq[p_j]_r\}$.

From $k\geqslant 2$ and construction of r and G, we know, at least one node does not have input arc in G. Without loss of generality, we can assume $[p_1]_r$ is this node.

Now, we prove $[p_1]_r$ is non-empty siphon in N.

From the definition of r, $[p_1]_r$ is non-empty. If ${}^{\bullet}[p_1]_r=\varnothing$, then, from ${}^{\bullet}[p_1]_r=\varnothing\subseteq[p_1]_r^{\bullet}$, it can be deduced that $[p_1]_r$ is a non-empty siphon in N. Otherwise, $\forall t\in{}^{\bullet}[p_1]_r\exists p\in[p_1]_r$, such that $t\in{}^{\bullet}p$. As $|{}^{\bullet}t\cap H|\neq\varnothing$, there must exist $p'\in H$, such that $p'\in{}^{\bullet}t$ (i.e. $t\in p'^{\bullet}$). As $[p_1]_r$ has no input arc in G, it can be deduced that $p'\in[p_1]_r$. So, $t\in[p_1]_r^{\bullet}$. From the generality of t, we know ${}^{\bullet}[p_1]_r\subseteq[p_1]_r^{\bullet}$. Therefore, we can deduce that $[p_1]_r$ is also a non-empty siphon in N. This contradicts that H is minimal non-empty siphon in N. So, $k=1$, i.e. Lemma 3.1 holds: H is P-connected. □

Below, we prove that minimal non empty siphon can be described by net structure.

Theorem 3.1. Let $N=(P,T;F)$ be an AC net. $H\subseteq P$ is a siphon in N. H is a minimal siphon iff

(1) $\forall t\in H^{\bullet}$, $|{}^{\bullet}t\cap H|=1$;

(2) H is P-connected.

Proof. First, the sufficient condition is present:

Assume $H'\subseteq H$ is a non-empty siphon in N. $\forall p\in H$, $p'\in H'$, from condition (2), there must exist directed path from p to p'. From condition (1) and definition of siphon, $p\in H'$. From the generality of p, $H\subseteq H'$. So, $H=H'$, H is a minimal siphon.

Now, we give the proof of necessary condition:

First, we prove that condition (1) holds. H is a siphon in N, so $\forall t\in H^{\bullet}$, $|{}^{\bullet}t\cap H|>0$.

Assume $\exists t'\in H^{\bullet}$, $|{}^{\bullet}t'\cap H|\geqslant 2$. As N is an AC net, if ${}^{\bullet}t'=\{p_1,\ldots,p_m\}$, we can let $p_1^{\bullet}\subseteq p_2^{\bullet}\subseteq\ldots\subseteq$

$p_m^{\cdot}$, $p_1,\ldots,p_k \in {}^{\cdot}t'$ and $p_1,\ldots,p_k \in H$, $p_{k+1},\ldots,p_m \notin H$ $(k \geqslant 2)$.

Now we prove: $H_1 = H - \{p_1\}$ which is non-empty proper subset of H in N is also a siphon. $\forall t \in {}^{\cdot}H_1$, we have $t \in {}^{\cdot}H$, and so, $t \in H^{\cdot}$.

(a) If $t \in p_k^{\cdot}$, t must be an element in $H_1^{\cdot}$.

(b) If $t \notin p_k^{\cdot}$, then, as N is an AC net, $({}^{\cdot}t \cap {}^{\cdot}t') \cap H = \varnothing$, we have $t \cap H \subseteq H - {}^{\cdot}t' \subseteq H - \{p_1\} = H_1$, i.e. $t \in H_1^{\cdot}$.

From (a), (b) and the generality of t, ${}^{\cdot}H_1 \subseteq H_1^{\cdot}$. This contradicts that H is a minimal siphon. Therefore, condition (1) holds.

From Lemma 3.1, condition (2) holds. □

Lemma 3.2.[10] Let N be an AC net. If every (minimal) siphon in (N,M_0) contains a marked trap, then AC system (N,M_0) is live.

In the following, we give the proof of liveness monotonicity theorem in AC net, i.e. the sufficient and necessary condition of liveness monotonicity in AC net.

Theorem 3.2. Let $N=(P,T;F)$ be an AC net.

$\Sigma_0=(N,M_0)$ is live, and $\forall M_i$, $M_i \geqslant M_0$, $\Sigma_i=(N,M_i)$ is also live (i.e. liveness satisfies monotonicity) $\Leftrightarrow$ every (minimal) siphon in Σ_0 contains a marked trap.

Proof. First, we prove the sufficient condition:

As every siphon in (N,M_0) has a marked trap, then $\forall M_i$, $M_i \geqslant M_0$, every siphon in (N,M_i) must also contain a marked trap. From Lemma 3.2, we can deduce AC net (N,M_i) is live.

Now, we give the proof of the necessary condition:

We only need to prove: if there exists a minimal non-empty siphon H, such that every trap in it under M_0 is unmarked, then there exists $M \geqslant M_0$, (N,M) is not live.

We find the maximum trap in H by the following algorithm.

1. $H' \leftarrow H$, Σ' is a subsystem generated by H', $i \leftarrow 0$.
2. If $\exists\, t \in T'$, such that $t^{\cdot} = \varnothing$, let ${}^{\cdot}t = \{p\}$,
   then t is denoted by t_{i+1}, p is denoted by p_{i+1};
   else, stop.
3. Let $H' \leftarrow H' - \{p_i\}$, $\Sigma' = (H', T'; F', M'_0) \leftarrow$ subsystem generated by H', $i \leftarrow i+1$.
4. If $H' = \varnothing$, stop; else, goto 2.

In step 2, we let ${}^{\cdot}t = \{p\}$ just because of $|{}^{\cdot}t \cap H| = 1$. As $|H|$ is finite, the above procedure must stop at step 2 or 4. Let terminated value of i be m.

Because p_i which is found at step 2 does not belong to any trap, in the end, we get that H' is the maximum trap (probably empty).

Now, we seek an $M \geqslant M_0$ and a firing sequence σ, such that $M[\sigma > M'$ and H is unmarked in M'.

1. $i \leftarrow m$, $\sigma' \leftarrow \varnothing$, $M' \leftarrow M_0$, $M'_0 \leftarrow M_0$, $\Sigma' \leftarrow \Sigma$, $j \leftarrow 0$.
2. If $i = 0$, stop; else, goto 3.
3. While $\exists\, p \in {}^{\cdot}t_i$, $p \notin H$ and $M'(p) < M'(p_i)$ do
   $M'(p) \leftarrow M'(p)+1$, $M'_0(p) \leftarrow M'_0(p)+1$
   end
4. While t_i is enabled do
   fire t_i, $j \leftarrow j+1$
   end

5. From $M'[\overbrace{t_i \ldots t_i}^{j}>M_i$, compute M_i.

6. $\sigma' \leftarrow \sigma' \overbrace{t_i \ldots t_i}^{j}$, $i \leftarrow i-1$, $M' \leftarrow M_i$, $j \leftarrow 0$, goto 2.

After executing step 4, $M_i(p_i)=0$. From the selection of t_i, firing t_i can only add marking to $p_j (j<i)$, not add marking to $p_k (k \geqslant i)$ or $p \in H'$. So, $M'(p_i)=0$ and $M'(H')=0$, where $H-H'=\{p_i \mid i=1,\ldots,m\}$. Therefore, when the program stops, $M'(H)=0$, i.e. H is unmarked in M'. Let $M=M'_0$, $\sigma=\sigma'$. Here $M>M_0$, but (N,M) is not live. This contradicts the liveness monotonicity. Necessary condition holds. □

In the following, two simple examples are given to illustrate the above theorem.

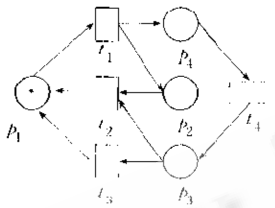


Fig. 1 An AC net, satisfying liveness monotonicity

Example 3.1. Consider the AC system (N,M_0), as shown by Fig. 1. It is easy to verify that (N,M_0) is live. In this net, there exists a minimal siphon $D=\{p_1,p_2,p_4\}$ and the siphon D contains a marked trap $\{p_1,p_3,p_4\}$. We can find $\forall M$. If $M \geqslant M_0$, then (N,M) must be live.

Example 3.2. Consider the AC system (N,M_0), as shown by Fig. 2. Also it is easy to verify that (N,M_0) is live. There exists a minimal siphon $D=\{p_1,p_2,p_3,p_4\}$ in this net, but the minimal siphon D does not contain marked trap. i.e. it does not satisfy the sufficient and necessary condition of Theorem 3.2. Obviously, if we add a marking to p_5, fire t_5, then all transitions in this net can't be fired any more, i.e. liveness does not have monotonicity.

Liveness monotonicity is a very strong condition. As we know, liveness of MG, SM and FC net satisfy liveness monotonicity. But, for AC net, from Theorem 3.2, we know that liveness monotonicity is equivalent to siphon-trap property. For general AC net, liveness does not satisfy monotonicity. This is why liveness of AC net is difficult to solve.

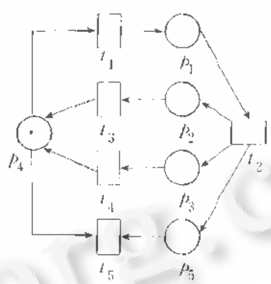


Fig. 2 An AC net, not satisfying liveness monotonicity

3.2 Liveness and safeness of AC nets

In this section, we give an equivalent condition for those AC nets which are live, safe and satisfy liveness monotonicity. First, we introduce some propositions.

Proposition 3.1. An AC net $N=(P,T;F)$ is live, safe and satisfies liveness monotonicity under $M_0 \Rightarrow$ every minimal siphon must be a trap.

Proof. For any minimal siphon H in N, from Theorem 3.2, H must contain a trap R. From Theorem 3.1, $\forall t \in R^{\cdot}$ and $|{}^{\cdot}t \cap R|=1$. From the definition of trap, $\forall t \in R^{\cdot}$, $|t^{\cdot} \cap R| \geqslant 1$. So, firing any transition in N can't decrease the marking of R.

If $H \neq R$, i.e. R is not a siphon, then there must exist a $t \in {}^{\cdot}R$ but $t \notin R^{\cdot}$. As N is live, t can be fired continuously, but firing t will monotonously increase the marking of H. This contradicts the safeness of N. Therefore, $H=R$. Proposition holds. □

Proposition 3.2. If an AC net N is live, safe and satisfies liveness monotonicity under M_0, then for every minimal siphon H, $|{}^{\cdot}t \cap H|=|t^{\cdot} \cap H|=1$.

Proof. From Proposition 3.1, every siphon H in N must be a trap, so $|t^{\cdot} \cap H \geqslant 1|$. From Theorem 3.1, $\forall t \in H$, $|{}^{\cdot}t \cap H|=1$.

Assume $\exists t$, $t^{\cdot} \cap H|>1$. As N is live under M_0, firing t will monotonously increase the marking of H. This contradicts the safeness of N. Therefore, $|{}^{\cdot}t \cap H|=|t^{\cdot} \cap H|=1$. Proposition holds. □

Proposition 3.3. If an AC net $N=(P,T;F)$ is live, safe and satisfies liveness monotonicity under M_0, then every $p\in P$ must be included in a minimal siphon.

Proof. Let $p\in P$ and p is not included in any minimal siphon. p at least has an input transition, otherwise, (N,M_0) is not live or there exist isolated nodes. Let $M_1\in R(M_0)$ and $M_1(p)$ be the maximum marking of p ((N,M_0) is safe). Consider the marking M'_1, such that $M'_1(p)=0$ and $M'_1(q)=M_1(p)$ $(q\neq p)$. As every minimal siphon contains a marked trap in M_1 (from Theorem 3.2), and p is not included in any siphon, any minimal siphon in N contains a marked trap in M'_1. From Lemma 3.2, (N,M'_1) is live, and a marking M_2' can be reached, such that $M'_2(p)\neq 0$.

We can define: $M_2(p)=M'_2(p)+M_1(p)$ and $M_2(q)=M'_2(q)$ $(q\neq p)$. M_2 must be included in $R(M_1)$. So, $M_2\in R(M_0)$. Therefore, $M_2(p)>M_1(p)$. But $M_1(p)$ is the maximum marking of p, this leads to contradiction. Proposition holds. □

The following theorem is based on the above three propositions and Theorem 3.2.

Theorem 3.3. An AC net $N=(P,T;F)$ is live, safe and satisfies liveness monotonicity under $M_0\Leftrightarrow$

(1) every minimal siphon H in N is a marked trap;

(2) $|{}^{\cdot}t\cap H|=|t^{\cdot}\cap H|=1$;

(3) $\forall\ p\in P$ must be included in a minimal siphon.

Proof. $\Leftarrow$ From (1) and Theorem 3.2, we know: N is live and satisfies liveness monotonicity.

Let $H_1,\ldots,H_n$ be all minimal siphons in N. From (1) and (2), we know that firing any transition in N will not change the marking of $H_1,\ldots,H_n$. Moreover, from (3), we deduce that $\forall\ p\in P$, $p\leqslant M_0(H_1)+\ldots+M_0(H_n)$. So, (N,M_0) is safe.

$\Rightarrow$ From Theorem 3.2 and Proposition 3.1, (1) holds.

From Proposition 3.2, (2) holds.

From Proposition 3.3, (3) holds. □

From above theorem, we know that the safeness and liveness monotonicity of AC net can be described by net structure. The following section provides a polynomial-time algorithm to verify whether AC net is live, safe and satisfies liveness monotonicity.

3.3 A polynomial-time algorithm for AC nets

An outline of the polynomial-time algorithm is given in this section. First, two propositions are given to support the algorithm.

Proposition 3.4.[9] Let N be a Petri Net.

N is structurally live and safe $\Rightarrow N$ is strongly connected.

Proposition 3.5. An AC net $N=(P,T;F)$ is live and satisfies liveness monotonicity under M_0.

(N,M_0) is safe $\Leftrightarrow N$ is covered by strongly-connected S-components.

Proof. Obviously, this proposition can be deduced from Theorem 3.3. □

Next, we give the outline of the algorithm (c.f. Refs.[6,11]):

Algorithm 1.

Input An AC system Σ.

Output Yes, Σ is live, safe and satisfies liveness monotonicity.

No, otherwise

(1) Check the net for being strongly connected.

If the net is not strongly connected stop with No.

(due to Proposition 3.4)

(2) For all places p find an S-component which contains p by

(2.1) finding a minimal siphon H containing p.

If p is not included in any minimal siphon H stop with No.

(due to Theorem 3.3)

(2.2) checking H for generating an S-component.

If H does not generate a subnet being an S-component

stop with No. (due to Proposition 3.5)

(3) For every pair (p,t) where p is a place such that $|p^{\cdot}|\geqslant 2$ and $t\in p^{\cdot}$,

(3.1) find a strongly connected siphon H containing p and no place in $t^{\cdot}$.

(3.2) If $H\neq\varnothing$, then find a minimal siphon H' containing p in H.

(3.2.1) If $H'\neq\varnothing$, then stop with "No" (due to Theorem 3.3)

(4) Check the existence of an unmarked siphon.

If an unmarked siphon exists then stop with "No"

else output "Yes".

For determining the worst case time complexity of algorithm, all steps are listed below (the following results can be found in Refs.[6,11]):

(1) Check the net for being strongly connected. $O(|P|+|T|+|F|)$

(2.1) Find a minimal siphon H containing p. $O(|T|(|P|+|T|+|F|))$

(2.2) Check H for generating an S-component. $O(|P||T|)$

(3.1) Find a strongly connected siphon H containing p and no place in $t^{\cdot}$. $O(|F||P|^2)$

(3.2) If $H\neq\varnothing$, then find a minimal siphon H' containing p in H. $O(|T||F|)$

(4) Check initial marking. $O(|P|^2|T|)$

Steps (1)(4) are called at most once. Step (2.1) and Step (2.2) are called at most $|P|$ times. Step (3.1) and Step (3.2) are called at most $|F|$ times. Thus, the worst case time complexity for the algorithm is given by

$$O(|P|+|T|+|F|+|P|(|T|(|P|+|T|+|F|)+|P||T|)+|P|^2|T|+|F|(|F||P|^2+|F||T|))$$
$$=O(|P||T||F|+|F|^2|P|^2+|F|^2|T|)$$

In the above, we have described a polynomial-time algorithm to decide whether an AC system is live, safe and having liveness monotonicity. This algorithm has a worst case time complexity of $O(|P||T||F|+|F|^2|P|^2+|F|^2|T|)$, which can be estimated as $O(n^6)$ (where $n=\max(|P|,|T|)$ and let $|P|=|T|=n$ and $|F|=n^2$).

Although we have given a polynomial-time algorithm to decide whether an AC net is live, safe and having liveness monotonicity, we still don't know what an AC net is, which satisfies liveness monotonicity. In the following two sections, two subclasses of AC nets that satisfy the liveness monotonicity are considered.

4 Strong I AC Nets

4.1 Related concepts and notations

In this section, Strong I AC net is considered.

Definition 4.1. For $t_1,t_2\in T$, if ${}^{\cdot}t_1\cap{}^{\cdot}t_2=\varnothing$, they are enabling independent.

Definition 4.2. Let (N,M_0) be a net. A siphon H in N is said to be controlled iff $\forall M\in R(M_0)$, $\exists p\in H$, $M(p)\geqslant 1$.

The new subclass of AC net, Strong I AC net, is given as follows.

Definition 4.3. Let $N=(P,T;F)$ be an AC net. N is called Strong I AC net iff:

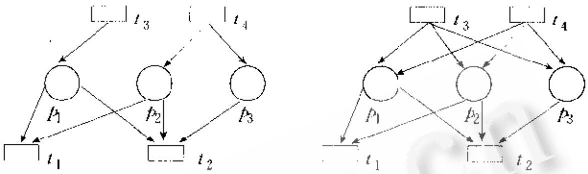
$$\forall (p,q)\in P\times P,\ p^{\cdot}\cap q^{\cdot}\neq\varnothing \quad\text{and}\quad p^{\cdot}\subset q^{\cdot}\Rightarrow {}^{\cdot}p={}^{\cdot}q.$$

Remark. Here the condition is equivalent to:

$\exists t\in T, t^{\bullet}=\{p_1,\dots,p_m\}$ and $p_1^{\bullet}\subseteq p_2^{\bullet}\dots\subseteq p_{l-1}^{\bullet}\subset p_l^{\bullet}\dots\subseteq p_{k-1}^{\bullet}\subset p_k^{\bullet}=\dots=p_m^{\bullet}\Rightarrow {}^{\bullet}p_1={}^{\bullet}p_2=\dots={}^{\bullet}p_m$.

In the following, some Strong I AC nets are shown.

Figure (4.1) (1) illustrates an AC net, but it is not a Strong I AC net (because $p_3^{\bullet}\subset p_1^{\bullet}=p_2^{\bullet}$, but p_1, p_2, p_3 have different pre-set). In Figure (4.1) (2), as $p_3^{\bullet}\subset p_1^{\bullet}=p_2^{\bullet}$ and p_1, p_2, p_3 have the same pre-set, i.e. ${}^{\bullet}p_1={}^{\bullet}p_2={}^{\bullet}p_3$, this net is a Strong I AC net. Figure (4.2) (1) illustrates a Free Choice net. Figure (4.2) (2) illustrates an Extended Free Choice net.



(1) AC net (2) Strong I AC net

Fig. 4.1

(1) Free choice net (FC net) (2) Extended free choice net (EFC net)

Fig. 4.2

Obviously, Free Choice nets and Extended Free Choice nets are included in Strong I AC nets, and Strong I AC nets are included in AC nets. This can be illustrated by Fig. (4.3). Therefore, Strong I AC nets are important subclass of AC nets, which could be applied in many practical cases.

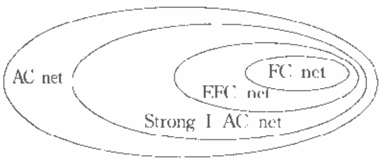


Fig. 4.3 Relation among AC nets, Strong I AC nets and EFC nets and FC nets

Corollary 4.1. Let $\Sigma_0=(P,T;F,M_0)$ be a Strong I AC system, satisfying the following.

If there exists $t\in T$, $t^{\bullet}=\{p_1,\dots,p_m\}$ and $p_1^{\bullet}\subseteq p_2^{\bullet}\subseteq\dots\subseteq p_{k-1}^{\bullet}\subset p_k^{\bullet}=\dots=p_m^{\bullet}=\{t_i,\dots,t_j\}$, then, for $p\in P$ and $p^{\bullet}\subset p_k^{\bullet}$, we have: $M_0(p)\geqslant\min(M_0(p_k),\dots,M_0(p_m))$.

Then the following conclusion holds:

$\forall M\in R(M_0)$, $t_i,\dots,t_j$ must be concurrently enabled or not enabled.

In the next three sections, we'll consider the liveness and safeness on Strong I AC net, and give a polynomial-time algorithm.

4.2 Liveness of Strong I AC nets[10]

In this section, a structural liveness theorem of Strong I AC nets is presented. First, a very important lemma is introduced.

Lemma 4.1. [8] An AC system (N, M_0) is live iff for all $M\in R(M_0)$ and (minimal) siphon H, there exists $p\in H$, satisfying $M(p)\geqslant 1$.

Remark. Corollary 27 in Ref. [8] considers the weighted system. Letting weight to be 1 will deduce this lemma.

Theorem 4.1. Let $\Sigma_1=(N, M_1)$ be a Strong I AC system, satisfying the condition of Lemma 4.1, then the liveness of Σ_1 is monotonic.

Proof. Assume $M_2\geqslant M_1$, (N, M_1) is live, but (N, M_2) is not live. Then, there must exist a siphon $D\subseteq P$ in M_2 and a firing sequence σ_2^1, such that $M_2[\sigma_2^1>M_2^1$ and D is unmarked in M_2^1(deduced by Lemma 4.1). If we can prove that: there exists a firing sequence σ_1^1, such that $M_1[\sigma_1^1>M_1^1$ and D is also unmarked in M_1^1, then this contradicts the liveness of Σ_1. The theorem holds.

In the following, we'll construct a firing sequence σ_1^1 and M_1^1. Let $\sigma_2^1=t_1\ldots t_n (n\geqslant 0)$.

Next, we'll operate by recurrence on the number of n to prove that there exist σ_1^1 and M_1^1.

Case 1. If $n=0$, then $M_2^1=M_2$. Let $\sigma_1^1=\varnothing$, then $M_1^1=M_1\leqslant M_2=M_2^1$. Obviously, D is unmarked in M_1. Theorem holds.

Case 2. Consider $n>0$. Let $t_1,\ldots,t_i$ are not enabled in M_1, but t_{i+1} is enabled in M_1. Then we have $(\bigcup_{j=1}^{i} {}^{\cdot}t_j)\cap {}^{\cdot}t_{i+1}=\varnothing$ from the definition of AC net and Corollary 4.1, i.e. $t_1,\ldots,t_i$ are enabling independent of t_{i+1}. So, we can rewrite $\sigma_2^1=t_{i+1}t_1\ldots t_i t_{i+2}\ldots t_n$, such that $M_1[t_{i+1}>M_1^2$ and $M_2[\sigma_2^1>M_2^1$.

We can repeat the above procedure on $t_1\ldots t_i t_{i+2}\ldots t_n$ and M_1^2. In the end, we can rewrite $\sigma_2^1=uv$, satisfying:

$$M_2[u>M_2^2[v>M_2^1 \tag{4.1}$$

$M_1[u>M_1^3$, $v=v_1\ldots v_m$, and any transition among $v_1,\ldots,v_m$ is not enabled in M_1^3.

Case 2.1. If $m=0$, then let $\sigma_1^1=u$, $M_1^1=M_1^3$. Theorem holds.

Case 2.2. If $m>0$, then from the liveness of Σ_1, there must exist $\omega=\omega_1\ldots\omega_l$, such that $M_1^3[\omega>M_1^4[v_1>M_1^5$. There are two cases as follows.

Case 2.2.1. If $(\bigcup_{i=1}^{m} {}^{\cdot}v_i)\cap(\bigcup_{j=1}^{l} {}^{\cdot}\omega_j)=\varnothing$, then ω is enabling independent of v. So, we have:

$$M_2^2[\omega>M_2^3[v_1>M_2^4[v_2\ldots v_m>M_2^5 \tag{4.2}$$

Obviously $M_1^5\leqslant M_2^4$. Next, we prove that:

$$(\bigcup_{i=1}^{l}\omega_i)\cap({}^{\cdot}D\cup D^{\cdot})=\varnothing \tag{4.3}$$

If (4.3) doesn't hold, then let $(\bigcup_{i=1}^{r}\omega_i)\cap({}^{\cdot}D\cup D^{\cdot})=\varnothing (r<l)$, but $\omega_{r+1}\in {}^{\cdot}D\cup D^{\cdot}$. Because ${}^{\cdot}D\subseteq D^{\cdot}$, we have $\omega_{r+1}\in D^{\cdot}$.

Let $d\in D$, $\omega_{r+1}\in d^{\cdot}$ and $M_1^3[\omega_1\ldots\omega_r>M_1^r$. From the above analysis, $M_1^r(d)>0$ and $M_1^3(d)-M_1^r(d)$. So, $M_1^3(d)>0$. As $M_2^2\geqslant M_1^3$, we have $M_2^2(d)>0$, but $M_2^1(d)=0$. From (4.1), there must exist v_i, such that $v_i\in d^{\cdot}$, i.e. ${}^{\cdot}v_i\cap {}^{\cdot}\omega_{r+1}\neq\varnothing$. This contradicts $(\bigcup_{i=1}^{m} {}^{\cdot}v_i)\cap(\bigcup_{j=1}^{l} {}^{\cdot}\omega_j)=\varnothing$. Therefore, (4.3) holds.

According to (4.2) (4.3), for all $d\in D$, there must have $M_2^5(d)=M_2^1(d)$, i.e. D is unmarked in M_2^5. As $m-1<n$, applying the recurrence hypothesis on M_1^5, M_2^4 and $v_2\ldots v_m$, there must exist σ'_1 and M', such that $M_1^5[\sigma'_1>M'$ and D is unmarked in M'. Let $\sigma_1^1=u\omega v_1\sigma'_1$, $M_1^1=M'$. Theorem holds.

Case 2.2.2. If $(\bigcup_{i=1}^{m} {}^{\cdot}v_i)\cap(\bigcup_{j=1}^{l} {}^{\cdot}\omega_j)\neq\varnothing$, then, let $(\bigcup_{i=1}^{m} {}^{\cdot}v_i)\cap(\bigcup_{j=1}^{k} {}^{\cdot}\omega_j)=\varnothing$ $(k<l)$, but $(\bigcup_{i=1}^{m} {}^{\cdot}v_i)\cap {}^{\cdot}\omega_{k+1}\neq\varnothing$. Moreover, let $(\bigcup_{i=1}^{r} {}^{\cdot}v_i)\cap {}^{\cdot}\omega_{k+1}=\varnothing$ $(r<m)$, but ${}^{\cdot}v_{r+1}\cap {}^{\cdot}\omega_{k+1}\neq\varnothing$. These imply $(\bigcup_{i=1}^{r} {}^{\cdot}v_i)\cap {}^{\cdot}v_{r+1}=\varnothing$ (from the definition of AC net). Therefore, v_{r+1} is enabling independent of $v_1,\ldots,v_r$.

Let $v'=v_{r+1}v_1\ldots v_r v_{r+2}\ldots v_m$, $\omega'=\omega_1\ldots\omega_k$. As ${}^{\cdot}v_{r+1}\cap {}^{\cdot}\omega_{k+1}\neq\varnothing$, and from Corollary 4.1 and the definition of AC net, we know that v_{r+1} is enabled in $M_1^{4'}$ $(M_1^3[\omega'>M_1^{4'})$. Then it is true that $M_2^2[v'>M_2^1$. Let $M_1^3[\omega'>M_1^{4'}[v_{r+1}>M_1^{5'}$, then ω' is enabling independent of v'. This transforms to Case 2.2.1. Theorem holds.

Up to now, we have proved that if Σ_1 is live, then $\forall\ M\geqslant M_1$, $\Sigma=(N,M)$ is live too, i.e. the liveness of Σ_1 is monotonic. □

Theorem 4.2. Let $N=(P,T;F)$ be a Strong 1 AC net. N is structurally live $\leftrightarrow$ every (minimal) siphon contains a trap.

Proof. Sufficient condition:

As every siphon contains a trap, there must exist a marking M, such that every (minimal) siphon contains a marked trap. From Lemma 3.2, it can be deduced that AC net is live in M. Therefore, N is structurally live.

Necessary condition:

As N is structurally live, there exists a marking M, such that $\Sigma=(N,M)$ is live. We construct a firing sequence σ_1, $M[\sigma_1>M_1$, such that the following conclusion holds in M_1:

If $\exists\, t\in T$, ${}^{\cdot}t=\{p_1,\dots,p_m\}$, $p_1^{\cdot}\subseteq p_2^{\cdot}\subseteq\dots\subseteq p_{k-1}^{\cdot}\subset p_k^{\cdot}=\dots=p_m^{\cdot}$, then $p\in P$ and $p^{\cdot}\subset p_m^{\cdot}$. We have $M(p)\geq\min(M_1(p_k),\dots,M_1(p_m))$.

Find the firing sequence by the following algorithm.

1. $T'\leftarrow T$; $\sigma\leftarrow\varnothing$; $\Sigma'=(P',T';F',M')\leftarrow\Sigma_1$; $T''\leftarrow T$; $i\leftarrow 1$.
2. If $\exists\, t\in T'$, let ${}^{\cdot}t=\{p_1,\dots,p_m\}$, and $p_1^{\cdot}\subset p_2^{\cdot}\subset\dots\subset p_{k-1}^{\cdot}\subset p_k^{\cdot}=\dots=p_m^{\cdot}$. For all $p'^{\cdot}\subset p_k^{\cdot}$, there is $M'(p')<\min(M'(p_k),\dots,M'(p_m))$, then t is denoted by t_i; else, stop.
3. For $j=1$ to $k-1$ do
   - 3.1. If $\exists\, M'(p_j)<\min(M'(p_k),\dots,M'(p_m))$, then find $t'\in p_k^{\cdot}$ and $p_j\notin{}^{\cdot}t'$; else, goto 3.4.
   - 3.2. If t' is enabled in M', then fire t', $\sigma\leftarrow\sigma t'$; $M'[t'>M''$, $M'\leftarrow M''$; goto 3.1.
   - 3.3. If $\Sigma'=(P',T'';F',M')$, find a firing sequence σ', such that t'' ($t''\in{}^{\cdot}p_k$) is enabled, then fire this firing sequence and t'', $\sigma\leftarrow\sigma\sigma' t''$, $M'[\sigma' t''>M''$, $M'\leftarrow M''$; goto 3.1.
   - 3.4. End.
4. While $\exists\, p\in P'$, $p^{\cdot}\subset p_k^{\cdot}$ and $p\notin\{p_1,\dots,p_{k-1}\}$, satisfying $M'(p)<\min(M'(p_k),\dots,M'(p_m))$ do
   - 4.1. Fire t_i, $\sigma\leftarrow\sigma t_i$, $M'[t_i>M''$.
   - 4.2. $M'\leftarrow M''$.
   - 4.3. End.
5. $T'\leftarrow T'-\{p_k^{\cdot}\}$; $i\leftarrow i+1$.
6. If $T'=\varnothing$, stop; else, goto 2.

In step 3.1, we must find a $t'\in p_k^{\cdot}$ and $p_j\notin{}^{\cdot}t'$, because $p_1^{\cdot}\subseteq p_2^{\cdot}\subseteq\dots\subseteq p_{k-1}^{\cdot}\subset p_k^{\cdot}=\dots=p_m^{\cdot}$. In step 3.3, as Σ' is live, there must exist a firing sequence σ', such that t'' ($t''\in{}^{\cdot}p_k$) is enabled. In step 4, after step 3, there must be $M'(p_1),\dots,M'(p_{k-1})\geq\min(M'(p_k),\dots,M'(p_m))$. Even if $M(p)=0$, we can also let t_i fire $\min(M'(p_k),\dots,M'(p_m))$ times, so in step 4, it can be reached that $M'(p)\geq\min(M'(p_k),\dots,M'(p_m))$.

After executing the above algorithm, we have σ and M'. Let $\sigma_1=\sigma$ and $M_1=M'$, then the conclusion mentioned above holds. From Theorem 4.1, it can be deduced that $\forall\, M$, if $M\geq M_1$, then (N,M) is live. So, from Theorem 3.2, we know that every siphon must contain a marked trap in M_1. Therefore, every (minimal) siphon in N contains a trap. Necessary condition holds. □

The above theorem is very similar to the liveness theorem for FC nets. Two simple examples are given to illustrate this theorem as follows:

Example 4.1. An AC net illustrated by Fig. (4.4) contains a minimal siphon $D=\{p_1,p_3\}$, and this siphon contains a trap $\{p_1,p_3\}$. If we put some tokens in p_1 or p_3, then this net is live, i.e. this net is structurally live.

Example 4.2. An AC net illustrated by Fig. (4.5) contains a minimal siphon $D=\{p_1,p_3\}$, but this siphon does not contain any trap. It is easy to verify that there does not exist any living marking.

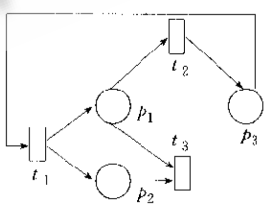


Fig. 4.4 Structurally live AC net

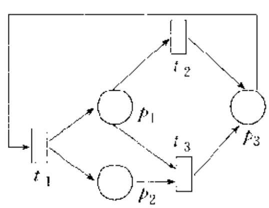


Fig. 4.5 Not structurally live AC net

From the liveness theorem of Strong I AC nets, someone may ask whether the following theorem holds: Strong I AC system $\Sigma=(N,M_0)$ is live iff every (minimal) siphon contains a marked trap.

Unfortunately, this proposition is wrong. The Strong I AC system in Fig. (4.6) gives a contrary example.

Figure (4.6) illustrates a live Strong I AC net. This net contains a minimal siphon H, $\{p_2, p_3, p_4\}$, and H contains a trap $\{p_3, p_4\}$. Although this trap is unmarked, this Strong I AC net is still live.

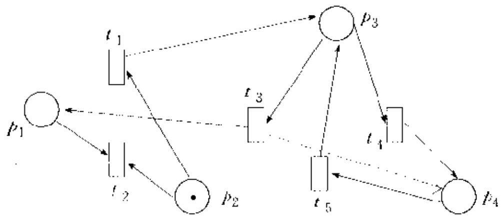


Fig. 4.6 A live Strong I AC net in which a siphon contains no marked trap

4.3 Safeness and liveness of Strong I AC nets

Safeness is another important property in Petri nets and it is discussed in this section.

Proposition 4.1. A Strong I AC net $N=(P,T;F)$ is structurally live and safe $\Rightarrow$ every minimal siphon must be a trap.

Proof. As N is structurally live, there must exist a living marking M. For any minimal siphon H in N, from Theorem 4.2, siphon H must contain a trap R. From Theorem 3.1, $\forall t \in R^{\cdot}$, $|^{\cdot}t \cap R| = 1$ and from the definition of trap: $\forall t \in R^{\cdot}$, $|t^{\cdot} \cap R| \geq 1$. So, firing any transition in N can't decrease the marking of R.

If $H \neq R$, i.e. R is not a siphon, then there must exist a $t \in {}^{\cdot}R$ but $t \notin R^{\cdot}$. As N is live, t can be fired continuously, but firing t will monotonously increase the marking of H. This contradicts the safeness of N. Therefore, $H=R$. Proposition holds. □

Proposition 4.2. If a Strong I AC net N is structurally live and safe, then for every minimal siphon H, $|^{\cdot}t \cap H| = |t^{\cdot} \cap H| = 1$.

Proof. From Proposition 4.1, every siphon H in N must be a trap, so $|t^{\cdot} \cap H \geq 1|$. From Theorem 3.1, $\forall t \in H$, $|^{\cdot}t \cap H| = 1$.

Assume $\exists$ t, $|t^{\cdot} \cap H| > 1$. As N is live, there must exist a living marking M. Firing t will monotonously increase the marking of H. This contradicts the safeness of N. Therefore, $|^{\cdot}t \cap H| = |t^{\cdot} \cap H| = 1$. Proposition holds. □

Lemma 4.2. Let $\Sigma = (N, M_0)$ be a Petri net system, H be a siphon in Σ. If H can be controlled in M_0, then $\forall$ M, such that $M(p) = M_0(p)$ $(p \in H)$ and $M(p) \leq M_0(p)$ $(p \notin H)$, H is controlled in M.

Proof. Assume H can't be controlled in M, then there must exist $M' \in R(M)$ and a firing sequence σ, such that $M\sigma > M'$ and $M'(H) = 0$.

Because $\forall$ $p \in N$, $M(p) \leq M_0(p)$, the firing sequence σ can also be fired in M_0 and a marking M'_0($M_0[\sigma > M'_0$) is reached. Firing σ will lead to $M'_0(H) = 0$ (because $\forall$ $p \in H$, $M(p) = M_0(p)$). This contradicts the condition. Lemma holds. □

Proposition 4.3. If $N=(P,T;F)$ is a structurally live and safe Strong I AC net, then every $p \in P$ must be included in a minimal siphon.

Proof. Let $p \in P$ and p is not included in any minimal siphon. p at least has one input transition, otherwise, N is not structurally live or there is an isolated node. As N is structurally live, there must exist a living marking M_0. Let $M_1 \in R(M_0)$ and $M_1(p)$ be the maximum marking of p (N is structurally safe).

Consider the marking M'_1, such that $M'_1(p) = 0$ and $M'_1(q) = M_1(p)$ $(q \neq p)$. As every minimal siphon can be controlled in M_1(from Lemma 4.1), and from Lemma 4.2, p is not included in any siphon, then any siphon in N can be controlled in M'_1. From Lemma 4.1, (N, M'_1) is live, and a marking M'_2 can be reached, such that $M'_2(p) \neq 0$.

We can define: $M_2(p) = M'_2(p) + M_1(p)$ and $M_2(q) = M'_2(q)$ $(q \neq p)$. M_2 must be included in $R(M_1)$. So, $M_2 \in R(M_0)$. Therefore, $M_2(p) > M_1(p)$, but $M_1(p)$ is the maximum marking of p. This leads to contradiction. Proposition holds. □

The following theorem is based on the above three propositions and Theorem 4.2.

Theorem 4.3. A Strong I AC net $N=(P,T;F)$ is structurally live and safe $\Leftrightarrow$

(1) every minimal siphon H in N is a trap;

(2) $|{}^{\bullet}t\cap H|=|t^{\bullet}\cap H|=1$;

(3) $\forall p\in P$ must be included in a minimal siphon.

Proof. $\Leftarrow$ From (1) and Theorem 4.2, N is structurally live.

We give a marking M of N. Let $H_1,\ldots,H_n$ be all minimal siphons in N. From (1) and (2), firing any transition in N will not change the marking of $H_1,\ldots,H_n$. Moreover, from (3), we deduce that $\forall p\in P$, $\dot{p}\leqslant M(H_1)+\ldots+M(H_n)$. So, (N,M) is safe. From the generality of M, N is structurally safe.

$\Rightarrow$ From Theorem 4.2 and Proposition 4.1, (1) holds.

From Proposition 4.2, (2) holds.

From Proposition 4.3, (3) holds. □

From Theorem 4.3, the structural safeness and structural liveness of Strong I AC net can be described by net structure. The following section provides a polynomial-time algorithm to verify if a Strong I AC net is structurally live and safe.

4.4 A polynomial-time algorithm

An outline of the polynomial-time algorithm is given in this section. Before the algorithm, a proposition is given.

Proposition 4.4. Let $N=(P,T;F)$ be structurally live. Strong I AC net N is structurally safe $\Leftrightarrow$ N is covered by strongly-connected S components.

Proof. Obviously, this proposition can be deduced from Theorem 4.3. □

Algorithm 2

Input A Strong I AC net N.

Output Yes, N is structurally live and safe.

No, otherwise.

(1) Check the net for being strongly connected.

If the net is not strongly connected stop with "No". (due to Proposition 3.4)

(2) For all places p find an S-component which contains p by

(2.1) finding a minimal siphon H containing p.

If p is not included in any minimal siphon H stop with "No". (due to Theorem 4.3)

(2.2) checking H for generating an S-component.

If H does not generate a subnet being an S-component, stop with "No". (due to Proposition 4.4)

(3) For every pair (p,t) where p is a place such that $|p^{\bullet}|\geqslant 2$ and $t\in p^{\bullet}$,

(3.1) find a strongly connected siphon H containing p and no place in $t^{\bullet}$.

(3.2) If $H\neq\varnothing$, then find a minimal siphon H' containing p in H.

(3.2.1) If $H'\neq\varnothing$, then stop with "No". (due to Theorem 4.3)

(4) stop and output "Yes".

This algorithm also has a worst case time complexity of $O(n^6)$ (where $n=\max(|P|,|T|)$, let $|P|=|T|=n$ and $|F|=n^2$).

5 Strong II AC Nets

5.1 Related concepts and notation

Definition 5.1. Let $N=(P,T;F)$ be an AC net. If $\exists\, t\in T$, ${}^\bullet t=\{p_1,\ldots,p_m\}$, $p_1^\bullet\subseteq p_2^\bullet\subseteq\ldots\subset p_k^\bullet=\ldots=p_m^\bullet$, let $P_{AC}={}^\bullet(p_m^\bullet)$, $T_{AC}=p_m^\bullet$, $F_{AC}=((P_{AC}\times T_{AC})\cup(T_{AC}\times P_{AC}))\cap F$, then we call $N_{AC}=(P_{AC},T_{AC};F_{AC})$ as an AC-component.

Definition 5.2. $p\in P$ is included in an AC-component iff there exists an AC-component $N_{AC}=(P_{AC},T_{AC};F_{AC})$ and $p\in P_{AC}$.

$t\in T$ is included in an AC-component iff there exists an AC-component $N_{AC}=(P_{AC},T_{AC};F_{AC})$ and $p\in T_{AC}$.

Definition 5.3. Let $N=(P,T;F)$ be an AC net. N is called a Strong II AC net iff:

$\forall\,(p,q)\in P\times P$, $(p^\bullet\cap q^\bullet\neq\varnothing$ and $p^\bullet\subset q^\bullet)\rightarrow\forall\, t_1,t_2\in q^\bullet$, $t_1^\bullet=t_2^\bullet$.

Figure (5.1) illustrates a general AC net, but it is not a Strong II AC net (because $p_3^\bullet\subset p_1^\bullet$, but t_3,t_4 have different post-sets). A Strong II AC net is shown in Fig. (5.2). In Fig. (5.2), as $p_2^\bullet\subset p_1^\bullet$ and t_3,t_4 have the same post-set, i.e. $t_3^\bullet=t_4^\bullet$, this net is a Strong II AC net. Figure (5.3) illustrates an Extended Free Choice net.

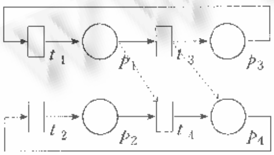


Fig. 5.1 General AC net

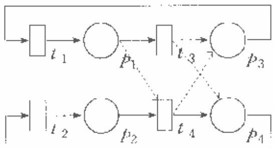


Fig. 5.2 Strong II AC net

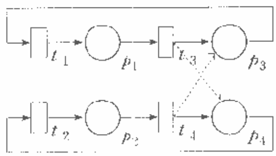


Fig. 5.3 Extended free choice net

It is easy to find that Extended Free Choice nets are included in Strong II AC nets, and Strong II AC nets are included in AC nets. This can be illustrated by Fig. (5.4). Therefore, Strong II AC nets are also an important subclass of AC nets which could be widely applied in many areas.

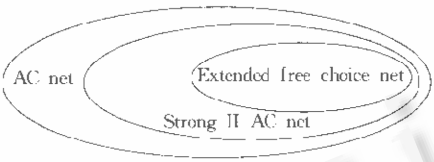


Fig. 5.4 Relation among AC nets, Strong II AC nets and EFC nets

5.2 Liveness of Strong II AC nets

In this section, a liveness theorem for Strong II AC net is presented.

Proposition 5.1. Let R be a trap in a Strong II AC net $N=(P,T;F)$. If $t\in$ AC-component and $t^\bullet\cap R=\varnothing$, then $\forall\, p\in{}^\bullet t$, $p^\bullet\nsubseteq{}^\bullet R$.

Proof. From Definition 5.3 and $t\in$ AC-component, $\forall\, t'\in p^\bullet$, $t'^\bullet=t^\bullet$. Moreover, $t^\bullet\cap R=\varnothing$, so $t'^\bullet\cap R=\varnothing$. Therefore, $t'\notin{}^\bullet R$. From the generality of t', $p^\bullet\nsubseteq{}^\bullet R$. Proposition holds. □

Algorithm 3. Find the maximum trap of a minimal siphon H in Strong II AC net.

1. Let $H'\leftarrow H$. $\Sigma'=(H',T',F',M'_0)\leftarrow$ system generated by H'.
2. If $\exists\, t\in T'$, such that $t^\bullet\cap H=\varnothing$ (let ${}^\bullet t=\{p\}$), then
   if $p\in$ AC-component, then denote $T_{i+1}=\{p^\bullet\}$, $A_{i+1}=\{p\}$;
   if $p\notin$ AC-component, then denote $T_{i+1}=\{t\}$, $A_{i+1}=\{p\}$;
   else stop.
3. Let $H'\leftarrow H'-A_i$, $\Sigma'=(H',T',F',M'_0)\leftarrow$ system generated by H', $i\leftarrow i+1$.
4. If $H'=\varnothing$, stop; else goto 2.

After Algorithm 3, assume $i=m$ and denote $A_{m+1}=H'$, $T_{m+1}={}^{\cdot}H'$, then A_{m+1} is a trap (maybe empty) which is needed. In step 2, let ${}^{\cdot}t=\{p\}$ be deduced from Theorem 3.1. In step 2, if $p\in$ AC-component, then, from Proposition 5.1, we know $p^{\cdot}\nsubseteq {}^{\cdot}R$ (R is a trap), so, denote $T_i=\{p^{\cdot}\}$. As $|H|$ is finite, the algorithm must be stopped at step 2 or step 4.

Corollary 5.1. Let $(A_i)_{i=1,m+1}$ be a partition of minimal siphon H in Strong II AC net by the above algorithm, then $\forall\ t\in T_i(i\leqslant m+1)$, firing t can only add markings to $A_j(j\leqslant i)$, not add marking to $A_k(k\geqslant i)$.

Lemma 5.1. Let $A\subseteq P$, $(A_i)_{i=1,m+1}$ be a partition of A, "$\prec$" be the order relation on the set markings of A defined by:

$$M'_A\prec M_A\Leftrightarrow\exists\ k\in[1,M]\,|\,\forall\ i>k,\quad M'(A_i)=M(A_i)\quad\text{and}\quad M'(A_k)<M(A_k).$$

Given an initial (finite) marking of A, M_A, there can not exist an infinite series of markings of A, starting from M_A and strictly decreasing with respect to $\prec$.

Proof. If such an infinite series exists, then either A is infinite, or A is not finitely marked. In both cases, we obtain a contradiction. □

Next, the sufficient and necessary condition of liveness on AC net is given.

Theorem 5.1. Let $\Sigma=(N,M_0)$ be a Strong II AC system, Σ is live $\Leftrightarrow$ every (minimal) siphon contains a marked trap.

Proof. $\Leftarrow$ From Lemma 3.2, it can be deduced that AC system is live.

$\Rightarrow$ Assume there exists a minimal siphon H, such that all traps are unmarked. From Algorithm 3, we can obtain a partition $(A_i)_{i=1,m+1}$ of H. As A_{m+1} is the maximum trap, we have $M(A_{m+1})=0$.

If $m=0$, then H is the trap. But H is unmarked, so Σ is not live. This contradicts the liveness of Σ. Therefore, in the following, we only consider $m>0$.

Let M_0^H be a marking of H under M_0. As Σ is live, there must exist a firing sequence σ (maybe $\sigma=\varnothing$), such that $M_0[\sigma>M_1$, *and* t_0(let t_0 be the first enabled transition in $H^{\cdot}$) is enabled in M_1.

Let $p_0\in{}^{\cdot}t_0\cap H$, $M_1[t_0>M_2$, M_2^H be a marking of H under M_2. Let $p_0\in A_i(i<m+1$, because A_{m+1} is a trap and unmarked).

1. If $p_0\in$ AC-component, then $t_0\in T_i$, firing t_0, we have $M_2^H\prec M_0^H$(from Corollary 5.1).
2. If $p_0\notin$ AC-component and $t_0\in T_i$, then, firing t_0, we have $M_2^H\prec M_0^H$(from Corollary 5.1).
3. If $p_0\notin$ AC-component and $t_0\notin T_i$, then, $\forall\ t\in T_i$, t and t_0 must be concurrently enabled. So, firing t, we also have $M_2^H\prec M_0^H$(from Corollary 5.1).

Σ is live, so according to the above method, we can construct an infinite series of markings of H, starting from M_0^H and strictly decreasing with respect to $\prec$. From Lemma 5.1, we know it is impossible. We obtain a contradiction. So every minimal siphon must contain a marked trap. Theorem holds. □

From this theorem, it is easy to deduce the following corollary.

Corollary 5.2. Let $N=(P,T;F)$ be a Strong II AC net. N is structurally live iff every minimal siphon contains a trap.

Two simple examples are given to illustrate Theorem 5.1:

Example 5.1. The Strong II AC system in Fig. (5.5) contains a minimal siphon $D=\{p_1,p_3\}$, and D contains a trap $\{p_1,p_3\}$. As p_3 is marked, then this system is live.

Example 5.2. The Strong II AC system in Fig. (5.6) contains a minimal siphon $D=\{p_1,p_3\}$, but D does not contain any marked trap. It is easy to verify that this system is not live.

5.3 Safeness and liveness of Strong II AC nets

From Theorem 5.1, Strong II AC net satisfies liveness monotonicity, so from Theorem 3.3, we can easily deduce the following theorem:

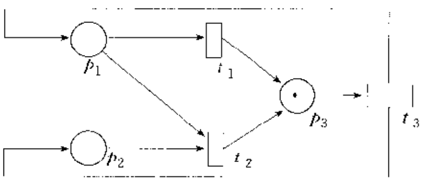


Fig. 5.5 A living Strong II AC system

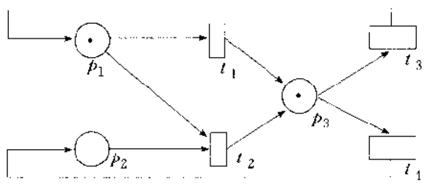


Fig. 5.6 A no living Strong II AC system

Theorem 5.2. A Strong II AC system $\Sigma=(N,M_0)$ is live and safe $\Leftrightarrow$

(1) every minimal siphon H in N is a marked trap;

(2) $|{}^{\cdot}t\cap H|=|t^{\cdot}\cap H|=1$;

(3) $\forall p\in P$ must be included in a minimal siphon.

Corollary 5.3. A Strong II AC net $N=(P,T;F)$ is structurally live and safe $\Leftrightarrow$

(1) every minimal siphon H in N is a trap;

(2) $|{}^{\cdot}t\cap H|=|t^{\cdot}\cap H|=1$;

(3) $\forall p\in P$ must be included in a minimal siphon.

Similar with the Strong I AC nets, the (structural) safeness and (structural) liveness of Strong II AC net can be described by net structure. The same polynomial time algorithm (see algorithm in Section 3.3) can be used to verify whether the Strong II AC net is live and safe.

6 Conclusions

In this paper, we have dealt with various issues related to the liveness and safeness of AC nets. Although the liveness and safeness of the general AC nets have not been solved completely, it presents a sufficient and necessary condition for those AC nets of which liveness has monotonicity and a polynomial time algorithm to decide whether a given AC system is live, safe and satisfies liveness monotonicity. Then two subclasses of AC nets (Strong I AC nets, Strong II AC nets) which have liveness monotonicity have been analyzed thoroughly. From the monotonicity of liveness on AC nets, we have learned why the general AC nets are difficult to solve. Moreover, this feature explains partially the non-monotonicity of the liveness on general Petri nets. As the liveness of general AC nets has not been solved completely, further work on these issues is planned. On the other hand, we have a conjecture that the AC nets which satisfy liveness monotonicity are the maximal subclass of Petri nets which can be decided in polynomial-time for liveness and safeness. This is a very challenging problem and also a direction that we plan to follow.

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论非对称选择网的活性与安全性

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摘要 活性与安全性是网系统的重要行为特性.该文的贡献在于为非对称选择网导出其活性与安全性的一般性质.文章讨论了活性具有单调性的非对称选择网活性与安全性的条件,并给出一个多项式时间算法来判定一个给定的非对称选择网是否是活的、安全的与活性满足单调性.文章还讨论了非对称选择网的两个子类(强化I型与强化II型),并给出活性满足单调性时其(结构)活与(结构)安全的充分条件.

关键词 非对称选择网,活,安全,结构活,结构安全,活性单调性,强化I型非对称选择网,强化II型非对称选择网.

中图法分类号 TP301