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Said-Bézier 型广义 Ball 曲线显式降多阶*

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Explicit Multi-Degree Reduction of Said-Bézier Generalized Ball Curves

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Abstract: This paper presents an optimal algorithm to compute multi-degree reduction of Said-Bézier generalized Ball curves (SBGB) with endpoints constraints in the L_2-norm. Based on the relations between Said-Bézier basis, Power basis and Jacobi basis, this paper deduces the explicit transformation matrix from SBGB basis to Jacobi basis and in reverse order. Then based on the inverse matrix of the above matrix and the orthogonality of Jacobi basis, an explicit constrained algorithm for multi-degree reduction of SBGB curves in the L_2-norm is put forward. This algorithm can be used in not only Said-Ball curve and Bézier curve but also the large class curves located between the two curves. This paper proves that the algorithm has some superiorities, including approximating optimal error of the degree reduction estimated beforehand, high order interpolation in the endpoints and multi-degree reduction in one time. Numerical examples demonstrate its validity and superiorities.
Key words: SBGB curve; Jacobi basis; optimal approximation; multi-degree reduction

摘 要: 给出了计算 Said-Bézier 型广义 Ball 曲线(SBGB 曲线)在 L_2 范数下保持端点约束的一种最佳降多阶算法.基于 SBGB 基函数、幂基函数和 Jacobi 基函数之间的相互转换关系,得到了 SBGB 基函数和 Jacobi 基函数之间的显式转换矩阵;进一步利用 Jacobi 基的正交性和上述转换矩阵的逆矩阵,导出了 SBGB 曲线在 L_2 范数下的显式约束降多阶算法.此算法蕴含了 Said-Ball 曲线、Bézier 曲线以及位置介于这两类曲线之间的一大类参数曲线的相应降多阶算法.证明了这是一种可以预报最佳误差且满足端点高阶约束的一次性降多阶算法.最后用数值实例说明了算法的正确性和优越性.
关键词: SBGB 曲线;Jacobi 基;最佳逼近;降多阶
中图法分类号: TP391 文献标识码: A

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1 Introduction

Ball curve is cubic polynomial curve introduced by Ball. A. A.[1] in the famous aircraft design system CONSURF in 1974. After that, there were a lot studies about Ball curve's high generalization and its properties. In the late 1980s, two different generalized Ball curves of arbitrary degree or arbitrary odd degree were obtained by Wang[2] and Said[3]; Hu, *et al*.[4] perfected Said's result to arbitrary degree and gave these curves' names: Wang-Ball curves and Said-Ball curves. In 2000, Wu[5] defined two family curves with a position parameter located between Wang-Ball curves and Said-Ball curves or Said-Ball curves and Bézier curves. These are Wang-Said generalized Ball curve and Said-Bézier generalized Ball curve. It has been proved that the Generalize Ball curves do not only possess many common properties of Bézier curves, but also are better in recursive evaluation, degree elevation and reduction algorithms[6−8].

Degree reduction of parameter curves is one of the most important operations in Computer-Aided Geometric Design (CAGD)[9]. It is widely used in data transfer and exchange between various CAD systems, and required in piecewise linear approximation and compression of shape data information in computer graphics[6,10]. Therefore, it is significantly necessary to study degree reduction algorithms of SBGB curve with the properties of end-points constraints, optimal approximated and explicit multi-degree reduction in the L_2-norm.

To satisfy this requirement, this paper presents a novel algorithm for explicit multi-degree reduction of SBGB curves with endpoints constraints. The main steps of this paper are as follows: Firstly, transform the SBGB curve to the form under Jacobi basis. Secondly, obtains the optimal multi-degree reduction approximation by applying orthogonality of Jacobi basis to the curve. Finally use the inversion matrix to get the degree-reduction SBGB curve. This paper proves that this optimal multi-degree reduction algorithm satisfies end-points high degree interpolation, explicit representation and the error predicted function. Moreover, according to the definition of the SBGB curve, obviously, when the position parameter K is equal to $0,\lfloor n/2\rfloor;1,2,\dots,\lfloor n/2\rfloor-1$, through this algorithm we can naturally get respectively the corresponding degree reduction algorithms of Said-Ball curve, Bézier curve and the large class curves located in between.

2 SBGB Curve and Jacobi Basis

Definition 1. Let $n\in N$, $n>2$, Said-Bézier generalized Ball curves with the given position parameter K be defined:

$$\boldsymbol{P}(t;n,K)=\sum_{i=0}^{n}\alpha_i(t;n,K)\boldsymbol{p}_i,\ 0\le t\le 1,$$

where $\boldsymbol{p}_i (i=0,1,\dots,n)$ are control points, $\{\alpha_i(t;n,K)\}_{i=0}^{n}$ are SBGB basis, defined as

$$\alpha_i(t;n,K)=\begin{cases}\dbinom{\lfloor n/2\rfloor+K+i}{i}t^i(1-t)^{\lfloor n/2\rfloor+K+1}, & 0\le i\le\lceil n/2\rceil-K-1\\ \dbinom{n}{i}t^i(1-t)^{n-i}, & \lceil n/2\rceil-K\le i\le\lfloor n/2\rfloor,\\ \alpha_{n-i}(1-t;n,K), & \lfloor n/2\rfloor+1\le i\le n\end{cases}$$

in which $\lfloor n/2\rfloor$ is the maximum integer not more than $n/2$, and $\lceil n/2\rceil$ is the minimum integer not less than $n/2$. From the definition, it's obvious to see that $\boldsymbol{P}(t;n,0)$ and $\boldsymbol{P}(t;n,\lfloor n/2\rfloor)$ are Said-Ball curve and Bézier curve respectively. When $1\le K\le\lfloor n/2\rfloor-1$, the curve $\boldsymbol{P}(t;n,K)$ is located between them.

A Jacobi polynomial basis $J_n^{(s,v)}(x)$ of degree n is an orthogonal polynomial on the weight function

$\rho(x)=(1-x)^s(1+x)^v,\ x\in[-1,1],(s>-1,v>-1)$, that is

$$\int_{-1}^{1}(1-x)^s(1+x)^v J_n^{(s,v)}(x)J_m^{(s,v)}(x)\mathrm{d}x=\begin{cases}0, & n\neq m\\ \delta_n^{(s,v)}, & n=m\end{cases} \quad (1)$$

where

$$\delta_n^{(s,v)}=\frac{2^{s+v+1}}{2n+s+v+1}\frac{(n+1)(n+2)...(n+v)}{(n+s+1)(n+s+2)...(n+s+v)} \quad (2)$$

3 The Conversion of SBGB Basis, Power Basis and Jacobi Basis

In this section, we discuss the transformation matrix between Jacobi basis and power basis, SBGB basis and power basis, and power basis and SBGB basis by three lemmas.

Lemma 1. Matrix $A_{(n-v+1)\times(n-m)}^{n,m,s,v}=\begin{pmatrix}A_{(m-v+1)\times(n-m)}^{n,m,s,v}\\ A_{(n-m)\times(n-m)}^{n,m,s,v}\end{pmatrix}$ can transform the Jacobi basis $\{J_i^{(2s,2v)}(x)\}_{i=m-s-v+1}^{n-s-v}$ with the weigh function $(1-x)^s(1+x)^v(s\geq 0,v\geq 0,-1\leq x\leq 1)$ to the combination of power basis $\{t^i\}_{i=v}^{n}$, i.e.

$$(1-x)^s(1+x)^v(J_{m-s-v+1}^{(2s,2v)}(x),J_{m-s-v+2}^{(2s,2v)}(x),...,J_{n-s-v}^{(2s,2v)}(x))=2^{s+v}(t^v,t^{v+1},...,t^n)A_{(n-v+1)(n-m)}^{n,m,s,v},\ 0\leq t=(1+x)/2\leq 1,$$

where

$$A_{(n-v+1)\times(n-m)}^{n,m,s,v}=\begin{pmatrix}A_{(m-v+1)\times(n-m)}^{n,m,s,v}\\ A_{(n-m)\times(n-m)}^{n,m,s,v}\end{pmatrix}=\begin{pmatrix}c_0^{m-s-v+1} & c_0^{m-s-v+2} & \cdots & c_0^{n-s-v-1} & c_0^{n-s-v}\\ \vdots & \vdots & \cdots & \vdots & \vdots\\ c_{m-v+1}^{m-s-v+1} & c_{m-v+1}^{m-s-v+2} & \cdots & c_{m-v+1}^{n-s-v-1} & c_{m-v+1}^{n-s-v}\\ 0 & c_{m-v+2}^{m-s-v+2} & \cdots & c_{m-v+2}^{n-s-v-1} & c_{m-v+2}^{n-s-v}\\ 0 & 0 & \cdots & c_{m-v+3}^{n-s-v-1} & c_{m-v+3}^{n-s-v}\\ \vdots & \vdots & \vdots & \vdots & \vdots\\ 0 & 0 & \cdots & 0 & c_{n-v}^{n-s-v}\end{pmatrix},$$

$$c_j^l=\begin{cases}\sum_{i=0}^{l}(-1)^{i-j}\binom{l+2s}{l-i}\binom{l+2s+2v+i}{i}\binom{i+s}{j}, & 0\leq j\leq s\\ \sum_{i=j-s}^{l}(-1)^{i-j}\binom{l+2s}{l-i}\binom{l+2s+2v+i}{i}\binom{i+s}{j}, & s+1\leq j\leq l+s\end{cases},$$

$$l=m-s-v+1,m-s-v+2,...,n-s-v.$$

Proof: By the deduction in the paper written by Zhang, *et al*.[11], the above conversion matrix is easy to obtain.

Lemma 2. Matrix $E_{(n+1)(n+1)}^{n}=(E_{(n+1)(m+1)}^{n,m},E_{(n+1)(n-m)}^{n,m})=(e_{i,j})_{(n+1)(n+1)}$ can transform SBGB basis to power basis $\{t^i\}_{i=0}^{n}$, i.e.

$$(\alpha_0(t;n,K),\alpha_1(t;n,K),...,\alpha_n(t;n,K))=(1,t,...,t^n)(E_{(n+1)(n+1)}^{n})^T,$$

Where

$$e_{i,j}=\begin{cases}(-1)^{j-i}\binom{\lfloor n/2\rfloor+K+i}{i}\binom{\lfloor n/2\rfloor+K+1}{j-i}, & 0\leq i\leq\lceil n/2\rceil-K-1, i\leq j\leq\lfloor n/2\rfloor+K+1+i\\ (-1)^{j-i}\binom{n}{i}\binom{n-i}{j-i}, & \lceil n/2\rceil-K\leq i\leq\lfloor n/2\rfloor+K, i\leq j\leq n\\ (-1)^{j-(\lfloor n/2\rfloor+K+1)}\binom{\lfloor n/2\rfloor+K+n-i}{n-i}\binom{n-i}{j-(\lfloor n/2\rfloor+K+1)}, & \begin{array}{l}\lfloor n/2\rfloor+K+1\leq i\leq n,\\ \lfloor n/2\rfloor+K+1\leq j\leq\lfloor n/2\rfloor+K+1+n-i\end{array}\\ 0, & \text{else}\end{cases}.$$

Proof: We just prove the part of matrix when $0\leq i\leq\lceil n/2\rceil-K-1$. It is easy to prove other cases in a similar way.

$$\alpha_i(t;n,K)=\binom{\lfloor n/2\rfloor+K+i}{i}t^i(1-t)^{\lfloor n/2\rfloor+K+1}=\binom{\lfloor n/2\rfloor+K+i}{i}t^i\sum_{j=0}^{\lfloor n/2\rfloor+K+1}\binom{\lfloor n/2\rfloor+k+1}{j}(-1)^j t^j$$
$$=\sum_{j=i}^{\lfloor n/2\rfloor+K+1+i}(-1)^{j-i}\binom{\lfloor n/2\rfloor+K+i}{i}\binom{\lfloor n/2\rfloor+k+1}{j-i}t^j.$$

The vector form of the above equation is what we need, so Lemma 2 holds.

Lemma 3. Matrix $G^n_{(n+1)(n+1)}=\begin{pmatrix}G^{n,v}_{v(n+1)}\\ G^{n,v}_{(n-v+1)(n+1)}\end{pmatrix}=(g_{i,j})_{(n+1)(n+1)}$ can transform power basis of degree n to SBGB basis, that is

$$(1,t,...,t^n)=(\alpha_0(t;n,K),\alpha_1(t;n,K),...,\alpha_n(t;n,K))(G^n_{(n+1)(n+1)})^T,$$

where

$$g_{i,j}=\begin{cases}\binom{\lfloor n/2\rfloor+K+j-i}{j-i}\Big/\binom{\lfloor n/2\rfloor+K+j}{j}, & 0\le j\le\lceil n/2\rceil-K-1, 0\le i\le j\\ \binom{n-i}{j-i}\Big/\binom{n}{j}, & \lceil n/2\rceil-K\le j\le\lfloor n/2\rfloor, 0\le i\le j\\ \left(\sum_{s=0}^{n-j}(-1)^s\binom{n-s}{n-j-s}\binom{i}{s}\right)\Big/\binom{n}{j}, & \lfloor n/2\rfloor+1\le j\le\lfloor n/2\rfloor+K, 0\le i\le n\\ \left(\sum_{s=0}^{n-j}(-1)^s\binom{n-s}{n-j-s}\binom{i}{s}\right)\Big/\binom{\lfloor n/2\rfloor+K+n-j}{n-j}, & \lfloor n/2\rfloor+K+1\le j\le n, 0\le i\le n\\ 0, & \text{else}\end{cases}.$$

Proof: In order to prove the lemma, we first introduce the dual functions of SBGB basis. Suppose $\{B_i(u)\}_{i=0}^n$ is a basis of the space of polynomials of degree non-exceeding n. Linear combination $\{\lambda_i B\}_{i=0}^n$ will be called the dual function of $\{B_i(u)\}_{i=0}^n$ if the following equations exist:

$$\lambda_i B_j=\delta_{ij}=\begin{cases}0, & i\ne j\\ 1, & i=j\end{cases}\quad i,j=0,1,...,n.$$

Let $f(t)=t^i(i=1,2,\ldots,n)$, according to the definition of the dual function of SBGB basis[12]:

$$\lambda_i f=\begin{cases}\binom{\lfloor n/2\rfloor+K+i}{i}^{-1}\sum_{s=0}^{i}\binom{\lfloor n/2\rfloor+K+i-s}{i-s}\frac{1}{s!}f^{(s)}(0), & 0\le i\le\lceil n/2\rceil-K-1\\ \binom{n}{i}^{-1}\sum_{s=0}^{i}\binom{n-s}{i-s}\frac{1}{s!}f^{(s)}(0), & \lceil n/2\rceil-K\le i\le\lfloor n/2\rfloor\\ \binom{n}{n-i}^{-1}\sum_{s=0}^{n-i}\binom{n-s}{n-i-s}(-1)^s\frac{1}{s!}f^{(s)}(1), & \lfloor n/2\rfloor+1\le i\le\lfloor n/2\rfloor+K\\ \binom{\lfloor n/2\rfloor+K+n-i}{n-i}^{-1}\sum_{s=0}^{n-i}\binom{\lfloor n/2\rfloor+K+n-i-s}{n-i-s}(-1)^s\frac{1}{s!}f^{(s)}(1), & \lfloor n/2\rfloor+K+1\le i\le n\end{cases}.$$

And the following two equations:

$$f^s(0)=\begin{cases}0, & t\ne i\\ i!, & t=i\end{cases},\quad f^s(1)=\begin{cases}0, & s>i\\ \binom{i}{s}s!, & s\le i\end{cases}.$$

We can easily accomplish the proof. Thus Lemma 3 holds.

4 End-Points Constraints Multi-Degree Reduction of SBGB Curves

Theorem 1. The optimal *n*–*m*(0<*m*<*n*) degree reduction of SBGB curve $p(t)=\sum_{i=0}^{n}\alpha_i(t;n,K)p_i$ in the L_2-norm with interpolation at the endpoints satisfying

$$\begin{aligned}&P^{(r)}(0;n,K)=\bar{P}^{(r)}(0;m,K), r=0,1,\ldots,v-1,\\&P^{(q)}(1;n,K)=\bar{P}^{(q)}(1;m,K), q=0,1,\ldots,s-1,\\&s+v\le m+2\end{aligned}$$

is $\bar{P}(t)=\sum_{i=0}^{m}\alpha_i(t;m,K)\bar{p}_i$, whose control points can be explicitly represented by the control points of the original curve as follows:

$$(\bar{p}_0,\bar{p}_1,\ldots,\bar{p}_m)^T=M^{n,m,s,v}_{(m+1)(n+1)}(p_0,p_1,\ldots,p_n)^T \tag{3}$$

where

$$M^{n,m,s,v}_{(m+1)(n+1)}=(G^{m}_{(m+1)(m+1)})^T(E^{n,m}_{(n+1)(m+1)})^T-(G^{m,v}_{(m-v+1)(m+1)})^T A^{n,m,s,v}_{(m-v+1)(n-m)}(A^{n,m,s,v}_{(n-m)(n-m)})^{-1}(E^{n,m}_{(n+1)(n-m)})^T \tag{4}$$

The exact error for this degree reduction approximation is

$$Er=\|P(t;n,K)-\bar{P}(t;m,K)\|_{L_2}=\sqrt{\frac{\sum_{i=m+1}^{n}\tilde{\boldsymbol{p}}^2_{i-s-v}\delta^{(2s,2v)}_{i-s-v}}{2^{2s+2v+1}}}\le\varepsilon^*,$$

where $\delta^{(2s,2v)}_{i-s-v},\tilde{\boldsymbol{p}}_{i-s-v}(i=m+1,m+2,\ldots,n)$, can be expressed respectively by Eq.(2) and the below equation:

$$(\tilde{\boldsymbol{p}}_{m-s-v+1},\tilde{\boldsymbol{p}}_{m-s-v+2},\ldots,\tilde{\boldsymbol{p}}_{n-s-v})^T=(A^{n,m,s,v}_{(n-m)(n-m)})^{-1}(E^{n,m}_{(n+1)(n-m)})^T(\boldsymbol{p}_0,\boldsymbol{p}_1,\ldots,\boldsymbol{p}_n)^T \tag{5}$$

Proof: Since $\boldsymbol{P}(t;n,K),\bar{\boldsymbol{P}}(t;m,K)$ have equal derived vector up to *v*−1(≥0)th and *s*−1(≥0)th orders, there exists an unknown polynomial $\tilde{\boldsymbol{P}}(t;n-s-v,K)$ of degree *n*–*s*–*v* such that

$$\boldsymbol{P}(t;n,K)-\bar{\boldsymbol{P}}(t;m,K)=(1-t)^s t^v\tilde{\boldsymbol{P}}(t;n-s-v,K),\ 0\le t\le 1 \tag{6}$$

By importing parameter transformation *t*=(*x*+1)/2(−1≤*x*≤1,0≤*t*≤1) in $\tilde{\boldsymbol{P}}(t;n-s-v,K)$ and denoting $\hat{\boldsymbol{P}}(x;n-s-v,K)$ as a polynomial function after transformation, we have the following equation

$$\boldsymbol{P}(t;n,K)-\bar{\boldsymbol{P}}(t;m,K)=(1-t)^s t^v\hat{\boldsymbol{P}}(2t-1;n-s-v,K),\ 0\le t\le 1 \tag{7}$$

Accordingly, that $\bar{\boldsymbol{P}}(t;m,K)$ is the optimal multi-degree reduced curve of $\boldsymbol{P}(t;n,K)$ in the L_2 norm equals that below Eq.(8) gives the minimum.

$$Er^2:=\|\boldsymbol{P}(t;n,K)-\bar{\boldsymbol{P}}(t;m,K)\|^2_{L_2}=\int_0^1(1-t)^{2s}t^{2v}\hat{\boldsymbol{P}}^2(2t-1;n-s-v,K)\mathrm{d}t \tag{8}$$

Since arbitrary polynomial can be uniquely expressed by a combination of Jacobi basis of the same order with the weight function $\omega(2t-1)=(1-t)^{2s}t^{2v}$[10], there is

$$\hat{\boldsymbol{P}}(2t-1;n-s-v,K)=\tilde{\boldsymbol{p}}_{n-s-v}J^{(2s,2v)}_{n-s-v}(2t-1)+\tilde{\boldsymbol{p}}_{n-s-v-1}J^{(2s,2v)}_{n-s-v-1}(2t-1)+\ldots+\tilde{\boldsymbol{p}}_0J^{(2s,2v)}_0(2t-1)$$

By Eq.(1), we have

$$\int_0^1(1-t)^{2s}t^{2v}J^{(2s,2v)}_n(2t-1)J^{(2s,2v)}_m(2t-1)\mathrm{d}t=\begin{cases}0, & n\ne m\\ \dfrac{\delta^{(2s,2v)}_n}{2^{2s+2v+1}}, & n=m\end{cases} \tag{9}$$

Considering the preceding three equations, we have

$$Er^2=\frac{\sum_{i=0}^{n-s-v}\tilde{\boldsymbol{p}}_i^2\delta^{(2s,2v)}_i}{2^{2s+2v+1}}.$$

Seeing both sides of Eq.(7), we know that $\{\tilde{\boldsymbol{p}}_i\}_{i=m-s-v+1}^{n-s-v}$ are decided by the original curve only and have no relation with the degree-reduced curve. So $\{\tilde{\boldsymbol{p}}_i\}_{i=0}^{m-s-v}$ being zero vectors can make Er^2 minimal. That is:

$$\boldsymbol{P}(t;n,K)-\overline{\boldsymbol{P}}(t;m,K)=(1-t)^s t^v \sum_{i=m-s-v+1}^{n-s-v} \tilde{\boldsymbol{p}}_i J_i^{(2s,2v)}(2t-1) \tag{10}$$

Accordingly, we just need to solve $\{\tilde{\boldsymbol{p}}_i\}_{i=m-s-v+1}^{n-s-v}$. To that end, firstly we apply Lemma 1 to error polynomial.

$$\begin{aligned}(1-t)^s t^v \hat{\boldsymbol{P}}(2t-1;n-s-v,K) &= (t^v,t^{v+1},\ldots,t^n)A_{(n-v+1)(n-m)}^{n,m,s,v}(\tilde{\boldsymbol{p}}_{m-s-v+1},\tilde{\boldsymbol{p}}_{m-s-v+2},\ldots,\tilde{\boldsymbol{p}}_{n-s-v})^T \\ &= (t^v,t^{v+1},\ldots,t^m)A_{(m-v+1)(n-m)}^{n,m,s,v}(\tilde{\boldsymbol{p}}_{m-s-v+1},\tilde{\boldsymbol{p}}_{m-s-v+2},\ldots,\tilde{\boldsymbol{p}}_{n-s-v})^T + \\ &\quad (t^{m+1},t^{m+2},\ldots,t^n)A_{(n-m)(n-m)}^{n,m,s,v}(\tilde{\boldsymbol{p}}_{m-s-v+1},\tilde{\boldsymbol{p}}_{m-s-v+2},\ldots,\tilde{\boldsymbol{p}}_{n-s-v})^T\end{aligned} \tag{11}$$

Secondly, we apply Lemma 2 to SBGB curve of degree n. It follows that

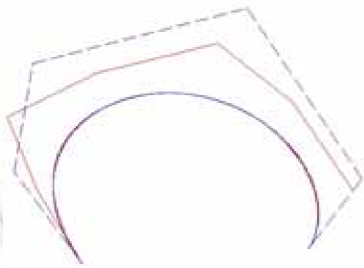
$$\begin{aligned}\boldsymbol{P}(t;n,K) &= \sum_{i=0}^{n}\alpha_i(t;n,K)\boldsymbol{p}_i = (1,t,\ldots,t^n)(E_{(n+1)(n+1)}^{n})^T(\boldsymbol{p}_0,\boldsymbol{p}_1,\ldots,\boldsymbol{p}_n)^T \\ &= (1,t,\ldots,t^m)(E_{(n+1)(m+1)}^{n,m})^T(\boldsymbol{p}_0,\boldsymbol{p}_1,\ldots,\boldsymbol{p}_n)^T + \\ &\quad (t^{m+1},t^{m+2},\ldots,t^n)(E_{(n+1)(n-m)}^{n,m})^T(\boldsymbol{p}_0,\boldsymbol{p}_1,\ldots,\boldsymbol{p}_n)^T\end{aligned} \tag{12}$$

At last by comparing the coefficients of $t^i(i\geq m+1)$ in Eq.(10), Eq.(11) and Eq.(12), we can obtain Eq.(5). By substituting Eq.(11), Eq.(12) into Eq.(10) and using Lemma 3, we can get Eq.(3) and Eq.(4). Therefore, Theorem 1 is proven.

5 Example Demonstration

In this example, we compare our method with the algorithm presented by Hu, *et al*.[13]. We use the solid line to express the original curves and control points, dash line the degree-reduced curves and corresponding control points, and dot line the parts of Hu, *et*. *al*[13].

Example 1. Given a SBGB curve of degree 7 with the position parameter K=2 whose corresponding control points are (420,500), (360,400), (320,300), (440,240), (600,200), (700,280), (780,400), (720,480). If 2-degree reduced curves hold at the two end points with C^0,C^1 continuities respectively, our method can compute the prior error: Er=0.855694 the control points of degree reduced curve are (420,500), (329.692,374.881), (354.252,227.329), (639.752,152.965), (792,384), (720,480). Also by using Hu's algorithm, we can conclude the control points of degree reduced curve are (420,500), (335.446,375.178), (339.554,227.396), (656.566,153.266), (792,384), (720,480), and the error bound is 24.1084 (see Fig.1).



(a) Our result

(b) Result of Hu's algorithm

Fig.1 SBGB curves (K=2), 2-degree reduced, two endpoints hold C^0,C^1 continuities respectively

6 Conclusion

This example illustrates the method in this paper is precise and effective. Functionally, the control points of degree-reduced curve can be expressed explicitly and the approximating error can be calculated beforehand which avoids multi-degree reduction in vain. For calculation, the operation of multi-degree reduction is simple. The matrix

$M_{(m+1)(n+1)}^{n,m,s,v}$ in theorem 1 can be calculated beforehand and stored in a database such that the reduction can be done in less time, which satisfies the requirement of modern CAD system. Therefore, our method has a good prospect of application.

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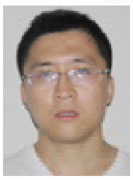

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