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基于模糊商空间的聚类分析方法*

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Cluster Analysis Based on Fuzzy Quotient Space

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Abstract: In this paper, on the basis of fuzzy quotient space theory, cluster analysis methods based on fuzzy similarity relations and normalized distance are proposed to solve data structure analysis of complex systems. Three conclusions are given: (1) the strictly clustering analysis theoretical description by introducing hierarchical structures of fuzzy similarity relation and normalized distance; (2) the effective and rapid clustering algorithms of their hierarchical structures; (3) a sufficient condition for isomorphic hierarchical structures. These conclusions are suitable to data structure analysis of all complex systems based on similarity relation.

Key words: fuzzy quotient space; hierarchical structure; cluster analysis; fuzzy similarity relation; normalized distance; isomorphism

摘　要: 在商空间理论基础上,提出了基于Fuzzy相似关系和归一化距离的聚类分析方法,用以解决复杂系统的数据结构分析问题.得到了如下结论:(1) 通过引入基于 Fuzzy 相似关系和归一化距离的分层递阶结构,建立了严格的聚类分析理论描述;(2) 给出了有效的分层递阶结构聚类的快速算法;(3) 给出了两个Fuzzy相似关系或由两个归一化距离诱导的 Fuzzy 相似关系是同构的充分条件.其中所研究的理论和方法适应于建立在相似关系之上的任何复杂系统的数据结构分析.

关键词: Fuzzy 商空间;分层递阶结构;聚类分析;Fuzzy 相似关系;归一化距离;同构

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1 Introduction

Since the fuzzy set theory was proposed in 1965 by L.A. Zadel, fuzzy techniques or methods have been applied

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to many fields extensively. Of them, fuzzy clustering technique, as a fundamental tool for revealing and analyzing structures, has been used frequently in actual application. Fuzzy cluster analysis is usually studied based on fuzzy equivalence relation[1−3], but, in general, cluster analysis based on fuzzy similarity relation[4−6] is more popular. This is because that fuzzy equivalence relation is difficult to verify, and the fuzzy similarity relation is easy to obtain. Basic fuzzy cluster analysis based on fuzzy similarity relation is based on transitive closure approach[1−3], whose procedure includes the following three steps.

- Step 1. Based on an actual problem, form a fuzzy similarity relation R;
- Step 2. Using transitive operation, obtain the transitive closure R^* of R, i.e. $R^*=t(R)$;
- Step 3. Obtain the final clustering result from R^*.

where R^* is a fuzzy equivalence relation. So the key to solve the question is transformed to compute the transitive closure of R, here it is also a difficult work.

In Refs.[4,5], He, *et al*. and Fu proposed some fuzzy clustering indirect methods based on fuzzy similarity relation, but the computational complexity is high. Besides, Hung, *et al*.[6] and Kamimura, *et al*.[7] proposed clustering methods based on distance, Tsekouras, *et al*.[8] also proposed a hierarchical fuzzy clustering approach.

Fuzzy quotient space theory was introduced by fuzzy equivalence relation in Ref.[9], and obtained the conclusion that any two of the fuzzy equivalence relation, the normalized equicrural distance and the hierarchical structure on space X are mutually equivalent, and get isomorphism and similarity principle between fuzzy equivalence relation R_1 and R_2. On one hand, those conclusions explain why similar results can be derived from various fuzzy equivalence relations on X. On the other hand, normalized equicrural distance and hierarchical structure, as the important researching methods, is introduced to fuzzy question researches. They are important because it is easy to accept the membership degree represented by the distance and structure of researching question when people study actual questions or learn knowledge.

In this paper, on the basis of Ref.[9], we propose cluster analysis theory based on fuzzy quotient space[10−12], and give direct clustering algorithms based on fuzzy similarity relation derived from normalized metric.

Definition 1.1[2]. Let $R\in F(X\times X)$, where $F(X\times X)$ denote all fuzzy sets on $X\times X$. If R satisfies

(1) $\forall x\in X$, $R(x,x)=1$;

(2) $\forall x,y\in X$, $R(x,y)=R(y,x)$, then R is called a fuzzy similarity relation on X.

If R is a fuzzy similarity relation on X, and satisfies:

(3) $\forall x,y\in X, R(x,y)\ge \sup_{z\in X}\{\min\{R(x,z),R(z,y)\}\}$, then R is called a fuzzy equivalence relation on X.

Definition 1.2[9]. Let R be a fuzzy equivalence relation on X. For any $\lambda\in[0,1]$, space $X(\lambda)$ consists of the equivalence class of cut relation R_λ (Note: R_λ is a crisp equivalence relation) such that $\forall\lambda_1,\lambda_2\in[0,1]$, $\lambda_1<\lambda_2\Rightarrow X(\lambda_1)<X(\lambda_2)$, so $\{X(\lambda)|0\le\lambda\le1\}$ consists of an ordering chain structure. Then $\{X(\lambda)|0\le\lambda\le1\}$ is called the hierarchical structure of R.

Definition 1.3[9]. Let R_1,R_2 be fuzzy equivalence relations on X, $\{X_1(\lambda)|0\le\lambda\le1\}$ and $\{X_2(\mu)|0\le\mu\le1\}$ are hierarchical structure of R_1 and R_2 respectively. If there exists a one-to-one mapping $f:[0,1]\to[0,1]$, and $f(x)$ is a strictly monotonic increasing function such that $\mu=f(\lambda)$. Then R_1 and R_2 is called isomorphism.

Lemma 1.1[2]. Let R be a fuzzy similarity relation on X, where X is a finite set, then

(1) $R\subseteq R^2$;

(2) $t(R)=\bigcup_{k=1}^{\infty}R^k$;

(3) $t(R)$ is fuzzy equivalence relation on X.

Lemma 1.2[9]. The following three statements are equivalent, i.e.:

(1) Given a fuzzy equivalence relation on X;

(2) Given a normalized equicrural distance on some quotient space of X;

(3) Given a hierarchical structure on X.

Lemma 1.3[9] (**Isomorphism Discrimination Theorem**). The following three statements are equivalent, i.e.

(1) Fuzzy equivalence relation R_1 and R_2 is called isomorphic;

(2) Let R_1 and R_2 be fuzzy equivalence relation on X. $\forall x,y,u,v\in X$,

$$R_1(x,y)<R_1(u,v)\leftrightarrow R_2(x,y)<R_2(u,v) \text{ and } R_1(x,y)=R_1(u,v)\leftrightarrow R_2(x,y)=R_2(u,v);$$

(3) There exists a one-to-one mapping f:[0,1]→[0,1], and $F(x)$ is a strictly monotonic increasing function such that $\forall x,y\in X$, $R_2(x,y)=F(R_1(x,y))$.

2 The Structure Representation of Fuzzy Clustering

In this section, we perform the research from fuzzy similarity relation on X, where X is a finite set.

Definition 2.1. Let R be a fuzzy similarity relation on X. For any $\lambda\in[0,1]$, R_λ denotes the cut relation of R.

$$D_\lambda=\{(x,y)\,|\,\exists x=x_1,x_2,...,x_m=y,\ni(x_i,x_{i+1})\in R,i=1,2,...,m-1\}.$$

Then D_λ is called the deriving relation from the base R_λ on X.

Proposition 2.1. The relation D_λ in Definition 2.1 is a crisp equivalence relation on X.

Theorem 2.1. Assume R is a fuzzy similarity relation on X, R_1 is a fuzzy similarity relation produced by the transitive closure of R, i.e. $R_1=t(R)$, its corresponding hierarchical structure is $\{X_1(\lambda)|0\le\lambda\le1\}$. $\{X_2(\lambda)|0\le\lambda\le1\}$ denotes the quotient space of D_λ in Definition2.1. Then $\forall\lambda\in[0,1]$, $X_1(\lambda)=X_2(\lambda)$.

Proof: $\forall\lambda\in[0,1]$

(1) If $(x,y)\in X_1(\lambda)$, i.e. $R_1(x,y)\ge\lambda$, we have $\lambda\le R_1(x,y)=\bigcup_{n=1}^{\infty}R^n(x,y)=\lim_{n\to\infty}R^n(x,y)$ by Lemma 1.1. Then, $\forall\varepsilon>0$, there exists a positive integer N such that $R^N(x,y)>R_1(x,y)-\varepsilon$, i.e.

$$\sup_{x_1,...,x_{N-1}\in X}\{R_1(x,x_1)\wedge R(x_1,x_2)\wedge...\wedge R(x_{N-1},y)\}>\lambda-\varepsilon \tag{2.1}$$

Therefore, there exists $y_1,\dots,y_{N-1}\in X$ such that

$$R_1(x,y_1)\wedge R(y_1,y_2)\wedge\dots\wedge R(y_{N-1},y)>\lambda-\varepsilon \tag{2.2}$$

In Eq.(2.2), let $\varepsilon\to0^+$, we have $R_1(x,y_1)\wedge R(y_1,y_2)\wedge\dots\wedge R(y_{N-1},y)\ge\lambda$, then $\exists x=y_0,y_1,...,y_N=y,\ni R(y_i,y_{i+1})\ge\lambda$, i.e. $(y_i,y_{i+1})\in R_\lambda$, $i=0,1,\dots,N-1$.

By Definition 2.1, we have $(x,y)\in X_2(\lambda)$, i.e. $X_1(\lambda)\subseteq X_2(\lambda)$.

(2) If $(x,y)\in X_2(\lambda)$, by Definition 2.1, $\exists x=x_0,x_1,...,x_m=y,\ni R(x_i,x_{i+1})\in R_\lambda$, i.e.

$$R(x_i,x_{i+1})\ge\lambda,i=0,1,...,m-1\Rightarrow R(x,x_1)\wedge R(x_1,x_2)\wedge...\wedge R(x_{m-1},y)\ge\lambda\Rightarrow$$
$$R_1(x,y)=\sup_n\{R^n(x,y)\}\ge R^m(x,y)\ge R(x,x_1)\wedge R(x_1,x_2)\wedge...\wedge R(x_{m-1},y)\ge\lambda$$

Then $(x,y)\in X_1(\lambda)$, i.e. $X_2(\lambda)\subseteq X_1(\lambda)$.

By (1) and (2), this theorem has been proved. □

Corollary 2.1. In Theorem 2.1, let R_2 be the fuzzy equivalence relation obtained by $\{X_2(\lambda)|0\le\lambda\le1\}$ as a corresponding hierarchical structure. Then $R_1=R_2$.

Proof: By Theorem 2.1, $\forall\lambda\in[0,1]$, $X_1(\lambda)=X_2(\lambda)\Rightarrow\forall\lambda\in[0,1]$, $R_{1\lambda}=R_{2\lambda}\Rightarrow R_1=R_2$. □

Corollary 2.2. Let R be a fuzzy similarity relation on X. Then $\forall\lambda\in[0,1]$, $[t(R)]_\lambda=t(R_\lambda)$.

Proof: We can directly obtain from the proof procedure of Theorem 2.1. □

Corollary 2.2 shows that the transitive closure operation and the cut relative operation on a fuzzy similarity relation are exchangeable. For a fuzzy similarity relation R, Theorem 2.1 shows that the hierarchical structure of its deriving equivalence relation of R is the same as the one of $t(R)$. Therefore, we may analyze the hierarchical structure of R from the hierarchical structure $\{D_\lambda|0\le\lambda\le1\}$ defined by Definition 2.1. On the other hand, Theorem 2.1 ensures that the algorithm in the following is reasonable.

Let $X=\{x_1,x_2,\dots,x_n\}$, R is a fuzzy similarity relation on X,

$D=\{R(x,y)|x,y\in X\}=\{\lambda_1,\lambda_2,\dots,\lambda_m\}$, where $1=\lambda_1>\lambda_2>\dots>\lambda_m$. Then the algorithm to obtain all the corresponding hierarchical structures of R is designed as follows.

Algorithm *A*.

- Step 1. $i\Leftarrow 0$;
- Step 2. $i\Leftarrow i+1$, $\lambda\Leftarrow\lambda_i$, $A\Leftarrow\{1,2,\dots,n\}$, $C\Leftarrow\varnothing$;
- Step 3. $B\Leftarrow\varnothing$;
- Step 4. $j\in A$, $B\Leftarrow B\cup\{x_j\}$, $A\Leftarrow A\backslash\{x_j\}$;
- Step 5. $\forall k\in A$, if $R(x_j,x_k)\ge\lambda$ then $B\Leftarrow B\cup\{x_k\}$, $A\Leftarrow A\backslash\{x_k\}$, $\forall s\in A$, if $R(x_k,x_s)\ge\lambda$ then $B\Leftarrow B\cup\{x_s\}$, $A\Leftarrow A\backslash\{x_s\}$, otherwise goto Step 6;
- Step 6. $C\Leftarrow BC\cup\{\{B\}\}$
- Step 7. If $A=\varnothing$, output $X(\lambda)=C$, then goto Step 8, otherwise goto Step 3.
- Step 8. If $i=m$ or $C=\{1,2,\dots,n\}$, then goto Step 9, otherwise goto Step 2.
- Step 9. End.

All clustering classes of R can be obtained from Algorithm A, and it is easy to perform the cluster analysis of R. Its computational complexity is not larger than $n\times(n-1)\times m/2$. Given $\forall\lambda\in[0,1]$, the computational complexity of getting $X(\lambda)$ is not larger than $n\times(n-1)/2$.

Example 1. Let $X=\{1,2,\dots,14\}$, R is a fuzzy similarity relation on X, whose matrix representation is given in the following.

By Algorithm A, we obtain the hierarchical structure of R as follows:

$X(1)=\{\{1\},\{2\},\dots,\{14\}\}$; $X(.9)=\{\{1,3\},\{2,4,5\},\{6\},\{7\},\dots,\{12\},\{11,13,14\}\}$;

$X(.8)=\{\{1,3\},\{2,4,5,6,7\},\{8\},\{9,10,12\},\{11,13,14\}\}$; $X(.7)=\{\{1,2,\dots,7\},\{8,9,10,12\},\{11,13,14\}\}$;

$X(.6)=X(.5)=\{\{1,2,\dots,10,12\},\{11,13,14\}\}$; $X(.4)=\{1,2,\dots,14\}$

The corresponding clustering map is presented in Fig.1.

$$R=\begin{bmatrix}
1 \\
.5 & 1 \\
.9 & .4 & 1 \\
.6 & .9 & .5 & 1 \\
.7 & .4 & .6 & .9 & 1 \\
.7 & .5 & .5 & .2 & .6 & 1 \\
.4 & .7 & .4 & .8 & .7 & .8 & 1 \\
.6 & .5 & .4 & .3 & .2 & .4 & .4 & 1 \\
.5 & .5 & .4 & .2 & .3 & .3 & .5 & .6 & 1 \\
.6 & .5 & .5 & .3 & .3 & .4 & .5 & .7 & .8 & 1 \\
.2 & .3 & .4 & .3 & .2 & .4 & .1 & .2 & .3 & .4 & 1 \\
.6 & .3 & .4 & .4 & .5 & .5 & .6 & .7 & .6 & .8 & .3 & 1 \\
.4 & .2 & .3 & .4 & .2 & .4 & .3 & .1 & .4 & .3 & .9 & .3 & 1 \\
.3 & .2 & .2 & .1 & .1 & .3 & .2 & .1 & .2 & .4 & .8 & .4 & .9 & 1
\end{bmatrix}$$

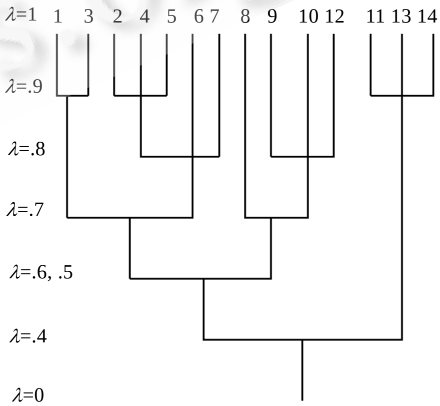

Fig.1 The clustering mapping of Example 1

3 Fuzzy Clustering Based on Normalized Distance

Lemma 1.2 shows that a fuzzy equivalence relation is equivalent to a normalized equicrural distance on some quotient space. In fact, the fuzzy clustering researches by introducing distance are usually attracting to researchers in fuzzy corresponding questions. From the geometric views, the length of distance may directly explain the membership degree, i.e. the shorter the distance between two elements, the bigger their relationship degree is. But to verify the equicrural condition of normalized distance is the same difficult as to verify condition (3) of a fuzzy equivalence relation, and it is easy to build a normalized distance on space X. Then, in this section, we discuss the relationship between the normalized metric on X and a fuzzy similarity relation on X, furthermore study the clustering questions based on normalized metric.

Definition 3.1. Let $d(\cdot,\cdot)$ be a normalized distance on X. Assume a one-to-one mapping f:[0,1]→[0,1], $f(\cdot)$ is a strictly monotonic decreasing function. We define a relation R on X as follows:

$\forall x,y\in X$, $R(x,y)=f(d(x,y))$, then $R(x,y)$ is called a fuzzy relation derived from d, where f is called the deriving mapping from d.

Proposition 3.1. The relation in Definition 3.1 is a fuzzy similarity relation on X.

The relation R in Definition 3.1 is also called a fuzzy similarity relation derived from distance d.

Theorem 3.1. $d(x,y)$ is a normalized distance on space $X\Leftrightarrow$ there is a fuzzy similarity relation R on X such that

$$R=f(d) \text{ and } \forall x,y\in X,\ f^{-}(R(x,y))\le \inf_{z\in X}\{f^{-}(R(x,z)), f^{-}(R(z,y))\} \tag{3.1}$$

where one-to-one mapping f:[0,1]→[0,1], $f(\cdot)$ is a strictly monotonic deceasing function and $f(0)=1$, f^{-} is the inverse function of f.

Proof: "⇒" By Definition 3.1 and Proposition 3.1, we may obtain a fuzzy similarity relation R on X, whose R is a deriving relation of d. Because $d(x,y)$ is a normalized distance on space X, therefore $\forall x,y\in X$, $d(x,y)\le d(x,z)+d(z,y)$, i.e. $d(x,y)\le \inf_{z\in X}\{d(x,z)+d(z,y)\}$, that is $\forall x,y\in X,\ f^{-}(R(x,y))\le \inf_{z\in X}\{f^{-}(R(x,z))+f^{-}(R(z,y))\}$.

"⇐" From $R=f(d)$ and satisfying conditions, we get $d=f^{-}(R(x,y))$. Because R is a fuzzy similarity relation R on X, we obtain conclusion as follows:

(1) $\forall x\in X$, $d(x,x)=f^{-}(R(x,x))=f^{-}(1)=0$;

(2) $\forall x,y\in X$, $d(x,y)=f^{-}(R(x,y))=f^{-}(R(y,x))=d(x,y)$;

(3) $\forall x,y,z\in X, d(x,y)=f^{-}(R(x,y))\le \inf_{z_1\in X}\{f^{-}(R(x,z_1))+f^{-}(R(z_1,y))\}$

$$\le f^{-}(R(x,z))+f^{-}(R(z,y))=d(x,z)+d(z,y);$$

(4) $\forall x,y\in X$, $0\le f^{-}(R(x,y))\le 1\Rightarrow \forall x,y\in X$, $0\le d(x,y)\le 1$.

Therefore $d(x,y)=f^{-}(R(x,y))$ is a normalized distance on space X. □

Corollary 3.1. Let $\forall x,y\in X$, $R(x,y)=1-d(x,y)$, then $d(x,y)$ is a normalized distance on space $X\Leftrightarrow$ is a fuzzy similarity relation on X and

$$\forall x,y\in X, 1+R(x,y)\ge \sup_{z\in X}\{R(x,z)+R(z,y)\} \tag{3.2}$$

Theorem 3.1 and Corollary 3.1 show that the concept of normalized distance on space X is stronger than the concept of fuzzy similarity relation on X, i.e. a given normalized distance on space X may be used to construct a fuzzy similarity relation on X, contrarily, the conclusion is not held, because a fuzzy similarity relation on X only satisfying condition (3.1) or (3.2) may be used to build a normalized distance on space X. Let $T(R)$ be the set of all fuzzy equivalence relations on X, $\mathfrak{R}_d$ denote the set of all fuzzy similarity relations derived from normalized metric d on X, $\mathfrak{R}$ denote the set of all fuzzy similarity relations on X, then they have the relationship as follows

$$T(R)\subset\mathfrak{R}_d\subset\mathfrak{R} \tag{3.3}$$

For convenience, we discuss only fuzzy similarity relations $R(x,y)=1-d(x,y)$, where R is derived from the normalized metric d. In fact, those conclusions in the following are also held for Definition 3.1.

Theorem 3.2. Assume $d(x,y)$ is a normalized distance on space, $R(x,y)=1-d(x,y)$, $B_\lambda=\{(x,y)|d(x,y)\le\lambda, 0\le\lambda\le1\}$. D_λ is the deriving crisp equivalence relation from the base B_λ on X. Let R_1 be a fuzzy equivalence relation, where $R_{1\lambda}=D_{1-\lambda}$ for any $\lambda\in[0,1]$, $R_2=t(R)$, then fuzzy equivalence relation R_1,R_2 satisfies $R_1=R_2$.

Proof: $\forall\lambda\in[0,1]$, $B_{1-\lambda}=\{(x,y)|d(x,y)\le1-\lambda\}=\{(x,y)|R(x,y)\ge\lambda\}$. By using Corollary 2.2,

$$R_{1\lambda}=D_{1-\lambda}=t(B_{1-\lambda})=t(R_\lambda)=[t(R)]_\lambda=R_{2\lambda}\Rightarrow R_1=R_2.$$

Definition 3.2. In Theorem 3.2, $X(\lambda)$ denotes the quotient space of $D_{1-\lambda}$, then $\{X(\lambda)|0\le\lambda\le1\}$ is called a hierarchical structure of X derived from the normalized distance d.

Corollary 3.2. In Theorem 3.2, let $\{X_1(\lambda)|0\le\lambda\le1\},\{X_2(\lambda)|0\le\lambda\le1\}$ be the hierarchical structures of R_1,R_2 respectively, then $\forall\lambda\in[0,1]$, $X_1(\lambda)=X_2(\lambda)$.

In fact, by Definition 3.2 and Corollary 3.2, $X_1(\lambda)$ is the quotient space of $D_{1-\lambda}$, i.e. $X_1(\lambda)=X(\lambda)$.

In the proof procedure of Theorem 3.2, we make only use of the normalized condition rather than the triangle law of distance. The conclusion in the following could directly be obtained from Theorem 3.2, Corollary 3.2, Definition 3.2 and Definition 1.2.

Theorem 3.3. Let d be a normalized distance on space X, $\forall x,y\in X$, $R(x,y)=1-d(x,y)$. Then, for any $\forall\lambda\in[0,1]$, the corresponding fuzzy clustering of R is uniquely determined by the quotient space of $D_{1-\lambda}$.

Let $X=\{x_1,x_2,\dots,x_n\}$, d is a normalized distance on space X, $D=\{R(x,y)|x,y\in X\}=\{d_1,d_2,\dots,d_m\}$, where $0=d_1<d_2<\dots<d_m$. Similar to Algorithm A in Section 2, we can also give the algorithm to obtain all the clustering classes of the fuzzy similarity relation derived from d.

Definition 3.3. Let R_1,R_2 be fuzzy similarity relation on X. If fuzzy equivalence relations derived from R_1,R_2 are isomorphic, then fuzzy similarity relations R_1 and R_2 is called isomorphism about fuzzy fuzzy clustering analysis.

Theorem 3.4. Assume d_1 and d_2 are normalized distances on space X, R_1 and R_2 are fuzzy similarity relations derived from d_1 and d_2 respectively. If there exists an one-to-one mapping f:[0,1]→[0,1] and $F(\cdot)$ is a strictly monotonic increasing function such that $d_2=F(d_1)$, then R_1 and R_2 are isomorphic.

Proof: R_1^0,R_2^0 denotes fuzzy equivalence relations derived from R_1 and R_2 respectively, their corresponding hierarchical structure is $\{X_1(\lambda)|0\le\lambda\le1\}$, $\{X_2(\lambda)|0\le\lambda\le1\}$ respectively. By Definition 1.3, $X_1(\lambda)$ and $X_2(\lambda)$ are quotient spaces of $R_{1\lambda}^0$ and $R_{2\mu}^0$ respectively. By Corollary 2.2, $R_{1\lambda}^0=t(R_{1\lambda})$, $R_{2\mu}^0=t(R_{2\mu})$.

$$\forall\lambda\in[0,1],\ R_{1\lambda}=\{(x,y)|R_1(x,y)\ge\lambda\}=\{(x,y)|d(x,y)\le1-\lambda\}=\{(x,y)|d_2(x,y)=F(d_1(x,y))\le F(1-\lambda)\}=R_{2f(\lambda)},$$

where $f(\lambda)=1-F(1-\lambda)$, it is obvious that f is a one-to-one mapping from [0,1] to [0,1] and $f(\cdot)$ is a strictly monotonic increasing function.

Then $R_{1\lambda}^0=t(R_{1\lambda})=t(R_{2f(\lambda)})=R_{2f(\lambda)}^0\Rightarrow X_1(\lambda)=X_2(f(\lambda))$, so R_1^0 and R_2^0 are isomorphic by Definition 1.3. Therefore, R_1 and R_2 are isomorphic about fuzzy clustering analysis by Definition 3.3.

Remark. The reverse of Theorem 3.4 is not held. Because "$R_{1\lambda}=R_{2f(\lambda)}\Rightarrow R_{1\lambda}^0=R_{2f(\lambda)}^0$" is held in the proof procedure of Theorem3.4, but its reverse proposition is not held.

Corollary 3.4. Assume R_1,R_2 are fuzzy similarity relations on X. If there exists an one-to-one mapping F:[0,1]→[0,1], $F(\cdot)$ is a strictly monotonic increasing function such that $\forall x,y\in X$, $R_2(x,y)=F(R_1(x,y))$, then R_1 and R_2 are isomorphic.

Theorem 3.4 and Corollary 3.4 state the fact that hierarchical structure is an inherent property of data structures for complex systems.

Example 2. Let $X\in R^n$, d_1 and d_2 are normalized distances on space, and $x,y\in X$,

$$d_1(x,y)=1-\exp(-\|x-y\|),\ d_2(x,y)=1-\exp(-\|x-y\|^2),$$

where $\|\cdot\|$ is a norm number of space X. Because d_1 and d_2 have the relationship as follows:

$$d_2=F(d_1)=\begin{cases}1-\exp(-\ln^2(1-d_1)), & d_1\in[0,1)\\ 1, & d_1=1\end{cases}.$$

And F satisfies the condition in Theorem 3.3, then fuzzy cluster analysis results derived from d_1 and d_2 are similar, their difference is only the selection of various distance thresholds.

4 Conclusions

In this paper, on the basis of fuzzy quotient space theory, we propose cluster analysis methods based on fuzzy similarity relations and normalized distance to solve data structure analysis of complex systems, and get three conclusions as follows: (1) the strictly clustering analysis theoretical description by introducing hierarchical structures of fuzzy similarity relation and normalized distance; (2) the effective and rapid clustering algorithms of their hierarchical structures; (3) a sufficient conditions for isomorphic hierarchical structures. These conclusions are suitable to data structure analysis of all complex systems based on similarity relation. At the same time these research works will be helpful to analyze effectively the structure of complicated problems and to solve real problems.

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References:

[1] Liang GS, Chou TY, Han TC. Cluster analysis based on fuzzy equivalence relation. European Journal of Operational Research, 2005,166:160−171.

[2] Wu CZ. Introduction of Fuzzy Set (I). Beijing: Beijing Normal University Press, 1984. 100−168 (in Chinese).

[3] Lee HS. An optimal algorithm for computing the max-min transitive closure of a fuzzy similarity matrix. Fuzzy Sets and Systems, 2001,123:129−136.

[4] He Q, Li HX, Shi ZZ, Lee ES. Fuzzy clustering method based on perturbation. Computers and Mathematics with Applications, 2003,46:929−946.

[5] Fu GY. Optimization methods for fuzzy clustering. Fuzzy Sets and Systems, 1998,93:301−309.

[6] Hung WL, Yang MS. Fuzzy clustering on lr-type fuzzy numbers with an application in Taiwanese tea evaluation. Fuzzy Sets and Systems, 2005,150:561−577.

[7] Kamimura H, Kurano M. Clustering by a fuzzy metric. Fuzzy Sets and Systems, 2001,120:249−254.

[8] Tsekouras G, Sarimveis H, Kavakli E, Bafas G. A hierarchical fuzzy-clustering approach to fuzzy modeling. Fuzzy Sets and Systems, 2005,150:245−266.

[9] Zhang L, Zhang B. The structure analysis of fuzzy sets. Int'l Journal Approx. Reason, 2005,40:92−108.

[10] Zhang B, Zhang L. The Theory and Applications of Problem Solving. Beijing: Tsinghua University Press, 1990. 35−86 (in Chinese).

[11] Zhang L, Zhang B. Fuzzy quotient space theory (fuzzy granule computing methods). Journal of Software, 2003,14(4):770−776 (in Chinese with English Abstract). http://www.jos.org.cn/1000-9825/14/770.htm

[12] Zhang L, Zhang B. Quotient space theory of problem solving. Fundamenta Informaticae, 2001,XXI:1001−1012.

附中文参考文献:

[2] 吴承忠.模糊集引论(上).北京:北京师范大学出版社,1989.100−168.

[10] 张钹,张铃.问题求解理论及应用.北京:清华大学出版社,1990.35−86.

[11] 张铃,张钹.模糊商空间理论(模糊粒度计算方法).软件学报,2003,14(4):770−776. http://www.jos.org.cn/1000-9825/14/770.htm

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第 3 届中国可信计算与信息安全学术会议

征 文 通 知

由解放军密码管理局和中国计算机学会容错专业委员会主办，中国人民解放军信息工程大学电子技术学院承办的“第 3 届中国可信计算与信息安全学术会议”将拟于 2008 年 10 月 25 日~28 日在河南郑州举行。会议将邀请本领域的专家作专题报告，将邀请研究人员、企业代表等，针对可信计算与信息安全领域的关键技术和热点问题进行交流和研讨。

1. 征文范围

会议重点征集可信计算与信息安全理论和技术方面的研究论文。具体包括（但不限于）:

（1）可信计算体系结构：可信计算理论，信任理论，可信计算平台体系结构，可信计算软件体系结构，可信网格，容错计算；

（2）可信软件：高可信软件，操作系统安全，数据库安全，软件容错，软件测试；

（3）可信硬件：可信计算平台，可信计算平台模块，信息安全芯片，智能卡，硬件容错，硬件测试，电子设备的物理安全；

（4）网络与通信安全：可信网络，网络安全技术，网络协议安全，网络容侵与容灾，通信安全，无线通信网络安全，计算机病毒技术；

（5）密码学：密码学的理论与技术，新型密码，密码应用技术；

（6）信息隐藏：信息隐藏，数字水印，数字版权管理；

（7）信息安全应用：电子政务安全，电子商务安全，可信计算与信息安全的应用，信息安全管理。

2. 征文要求

论文必须为未公开发表且未向学术刊物和其他学术会议投稿的最新研究成果，文稿使用中文或英文书写，字数一般不超过 6000。录用的英文稿件将在《武汉大学学报（英文版）》上发表，录用的中文稿件在核心期刊《武汉大学学报》（正刊）发表。

3. 重要日期

征文截止日期:	2008 年 4 月 30 日	录用通知:	2008 年 6 月 1 日
返回修改稿:	2008 年 6 月 20 日	定稿时间:	2008 年 7 月 1 日

4. 相关地址

本次会议投稿一律通过会议网站 http://www.tc2008.org 的“投稿系统”进行。

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