

基于参数分解的正则四边形插值细分曲面的快速求值*

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Rapid Evaluation of Regular Quad-Mesh Interpolatory Subdivision Surfaces Based on Parametric Decomposition

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Abstract: Two algorithms for evaluation of regular quad-mesh interpolatory subdivision surfaces are proposed. Algorithms are designed based on the parametric m -ary decomposition and construction of matrix sequence. The weights of the control points on the initial mesh can be obtained, through direct computation of the basic function values by multiplying the finite matrix sequence corresponding to the decomposition number sequence. Algorithm-I is based on 2D subdivision masks while the other is based on tensor-product. Numerical experiments show that the algorithms are efficient with low storage cost.

Key words: interpolatory subdivision scheme; evaluation; matrix sequence; parametric m -ary decomposition; tensor-product

摘要: 提出了两种正则四边形网格插值细分曲面的求值算法. 算法基于参数 m -进制分解和构造矩阵序列, 通过参数分解数列对应的矩阵乘积得到基函数值, 得到初始网格上对应控制点的权值, 从而实现插值细分曲面求值. 算法 1 基于 2D 细分掩模, 算法 2 基于张量积. 数值实验表明, 算法高效且低存储.

关键词: 插值细分格式; 求值; 矩阵序列; 参数 m -分解; 张量积

The recursive subdivision scheme produces a visually pleasing smooth surface in the limit by repeating refinement through a fixed set of rules on a user-specified control mesh. In general, it's difficult to evaluate an arbitrary point on the limit subdivision surface. But in many applications such as fitting, reparameterization and resampling, it is required to evaluate points on the subdivision surfaces at an arbitrary domain location.

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The study on evaluation of subdivision surfaces begins with J. Stam's work which focused on the analytic expression for Catmull-Clark and Loop subdivision surfaces^[1,2]. Both of above subdivision schemes are approximating, which are derived from bi-cubic B-spline and 3-direction quartic box-spline respectively. Afterwards some researchers have done more work on the evaluation of the subdivision surfaces generated by approximating schemes^[3-5] with the aid of eigenbasis functions, spline theory, special techniques around extraordinary points and etc.

The interpolatory subdivision scheme is widely used due to its behavior of preserving old vertices on the initial mesh. 1-4(binary) and 1-9(ternary) splitting schemes are two kinds of classical schemes. Butterfly scheme for triangular meshes (Dyn, *et al.*^[6]; Zorin^[7]) and Kobbelt's interpolatory scheme for quadrangular meshes (Kobbelt^[8]; Li, *et al.*^[9]) are motivated by a 4-point binary interpolatory subdivision scheme (Dyn, *et al.*^[10]). Ternary interpolatory subdivision scheme for triangular meshes (Hassan, *et al.*^[11]) and ternary subdivision scheme for quadrangular meshes (Li, *et al.*^[12]) are derived from an interpolatory subdivision for curves(Hassan, *et al.*^[13]). Unlike approximating schemes, the geometry of the limit surface obtained via interpolatory subdivision schemes does not have closed-form analytic expression even for a regular mesh, so it is very difficult to evaluate limit surfaces generated by the interpolatory schemes. In previous work we have presented an algorithm for evaluation of univariate interpolatory subdivision curves based on parametric m -ary decomposition and construction of matrix sequence. In this paper, an extended algorithm and a modified algorithm to evaluate interpolatory subdivision surfaces for regular quadrangular meshes will be discussed.

In Section 2, we describe the preliminary knowledge on the quad-mesh interpolatory subdivision schemes. Section 3 presents the detailed formulation of the algorithm for evaluating the regular quad-mesh interpolatory subdivision surfaces based on the 2D subdivision masks. Section 4 gives a modified algorithm based on the tensor-product behavior corresponding to the subdivision schemes. The numerical examples and results are presented in Section 5.

1 Preliminaries

Subdivision surfaces are defined by iteratively refining an initial mesh M^0 so that the sequence of increasingly faceted meshes M^1, M^2, \dots converge to some limit surface M^∞ . Each subdivision scheme S is associated with a mask $a = \{a_\alpha \in R : \alpha \in Z^s\}$, where $s=1$ in the curve case and $s=2$ in the surface case. The (stationary) subdivision scheme is a process which recursively defines a sequence of control points $P^k = \{p_\alpha^k : \alpha \in Z^s\}$ by a rule of the form with a mask $a = \{a_i\}_{i \in Z^s}$,

$$p_\alpha^{k+1} = \sum_{\beta \in Z^s} a_{\alpha-M\beta} p_\beta^k, \quad k \in \{0, 1, 2, \dots\} \quad (1)$$

where M is an $s \times s$ integer matrix such that $\lim_{n \rightarrow \infty} M^{-n} = 0$. The matrix M is called a dilation matrix. Binary (or dyadic) and ternary subdivision schemes are schemes with the matrices $M=2I$ and $M=3I$, respectively, where I is the $s \times s$ identity matrix. For the sake of simplicity, we only consider the symmetric stationary interpolatory schemes^[14,15]. Then we can get

Interpolatory: $a_0 = 1, a_{M\beta} = 0, \text{if } \beta \neq 0$;

Symmetric: $a_i = a_{-i}$. The width of support is $2N + 1$, such that $\beta \in \{-N, N\}$, where $N \in Z^s, M = mI, m \in Z_+$.

Let E be a complete set of representatives of the distinct cosets of Z^s / MZ^s . Then Z^s is the disjoint union of $\gamma + MZ^s, \gamma \in E$. Also, it is easy to know that the set $M^{-n}\alpha : \alpha \in Z^s, n \in N$ is dense in R^s .

In general, the subdivision scheme S here converges uniformly, that is

$$\sum_{\alpha \in Z^s} a_{\gamma - M\alpha} = 1, \gamma \in E \tag{2}$$

We take the ternary subdivision schemes as an example. For the univariate case, with a dilation matrix $M = m = 3$, Hassan, *et al.*^[13] have proposed the 4-point scheme

$$p_i^{k+1} = \sum_{j \in Z} a_{i-3j} p_j^k, i \in \{0,1,2\} \tag{3}$$

with the rule

$$\begin{aligned} p_{3i}^{k+1} &= p_i^k, \\ p_{3i+1}^{k+1} &= \sum_{j=-2}^1 a_{1+3j} p_{i-j}^k \\ p_{3i+2}^{k+1} &= \sum_{j=-2}^1 a_{2+3j} p_{i-j}^k. \end{aligned} \tag{4}$$

And the mask of the ternary subdivision scheme is $\{a_k\}_{-5}^5 = \{\alpha_3, \alpha_0, 0, \alpha_2, \alpha_1, 1, \alpha_1, \alpha_2, 0, \alpha_0, \alpha_3\}_{-5}^5$, where

$$\begin{aligned} \alpha_0 &= -\mu/6 - 1/18, \\ \alpha_1 &= \mu/2 + 13/18, \\ \alpha_2 &= -\mu/2 + 7/18, \\ \alpha_3 &= \mu/6 - 1/18. \end{aligned}$$

Hassan, *et al.*^[13] showed that the scheme is C^2 for $\frac{1}{15} < \mu < \frac{1}{9}$. For the regular quad-mesh, Li, *et al.*^[12] have proposed the subdivision scheme with a dilation matrix $M=3I, s=2$,

$$p_i^{k+1} = \sum_{j \in Z^2} a_{i-3j} p_j^k \tag{5}$$

where $i \in \{(0,0), (0,1), (0,2), (1,0), (1,1), (1,2), (2,0), (2,1), (2,2)\}$, with the mask $\{a_{ij}\} = \{a_i \cdot a_j\}$. Li, *et al.*^[12] showed that C^2 for $\frac{1}{15} < \mu < \frac{1}{9}$ in the regular case. Fig.1 illustrates the refinement process and control points associated with a given face F . Black points denote new face vertices, and gray points denote new edge vertices.

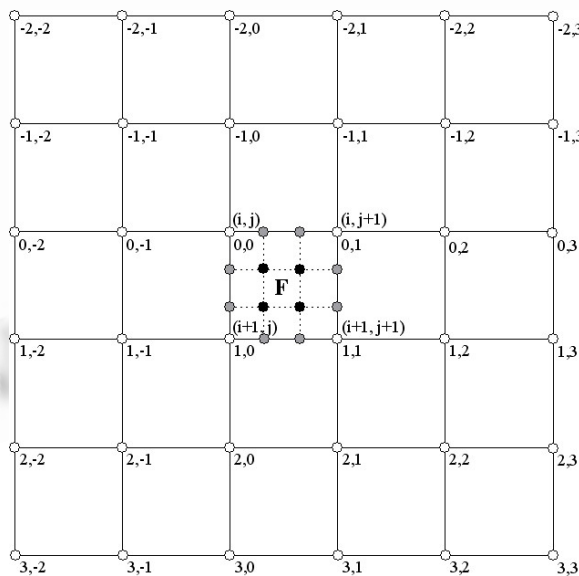


Fig.1 Coding method of the control point set P^0 associated with the given face F

The symmetric stationary subdivision scheme S with a mask a , satisfies the following refinement equation

$$\phi_s(t) = \sum_{-N}^N \alpha_j \phi_s(Mt - j) \quad (6)$$

where $N, j \in \mathbb{Z}^s, t \in \mathbb{R}^s$.

The limit basis function $\phi(t)$ associated with S can be obtained by refining the following vector with the mask $\{a\}$

$$\dots, (-N+1, 0), \dots, (-1, 0), (0, 1), (1, 0), \dots, (N, 0), \dots$$

Firstly, we consider the univariate case. For $2K$ points m -ary subdivision scheme S , which satisfies refinement equation (6) with $s = 1$, we define

$$L := 4K - 2 \quad (7)$$

And we construct square matrices of size L

$$T_l = (T_{ij})_{L \times L}, \quad T_{ij} = a_{mi-j+l+L(1-m)/2} \quad (8)$$

where $l = 0, 1, \dots, m-1$.

For the regular quad-mesh subdivision scheme S obtained from above univariate case, which satisfies refinement equation (6) with $s = 2$, we construct square matrices of size L^2

$$T_{l_1, l_2} = (T_{\eta\xi})_{L^2 \times L^2} \quad (9)$$

where $l_1, l_2 = 0, 1, \dots, m-1$. We decompose η and ξ as follows:

$$\eta = L * (i_1 - 1) + j_1; \quad \xi = L * (i_2 - 1) + j_2,$$

and denote vectors $(i_1, j_1), (i_2, j_2), (l_2, l_1), (L, L)$ by $\bar{\eta}, \bar{\xi}, \bar{l}, \bar{L}$ respectively, and then $(T_{l_1, l_2})_{\eta\xi}$ is defined as

$$(T_{l_1, l_2})_{\eta\xi} = a_{m\bar{\eta} - \bar{\xi} + \bar{l} + (1-m)\bar{L}/2} \quad (10)$$

We define the initial vector in the case of $s = 1$,

$$\Phi(t) = (\phi(-L/2+1+t), \dots, \phi(t), \dots, \phi(L/2+t))^T, \quad \Phi(0) = (\phi(-L/2+1), \dots, \phi(0), \dots, \phi(L/2))^T,$$

and in the case of $s = 2$, the initial vector $\Phi(t_1, t_2)$ of dimension L_2 is defined as

$$\begin{aligned} &(\phi(L_1+t_1, L_1+t_2), \phi(L_1+t_1, L_1+1+t_2), \dots, \phi(L_1+t_1, L_2+t_2), \phi(L_1+1+t_1, L_1+t_2), \dots, \\ &\phi(L_1+1+t_1, L_2+t_2), \phi(L_1+2+t_1, L_1+t_2), \dots, \phi(L_2+t_1, L_1+t_2), \phi(L_2+t_1, L_2+t_2))^T, \end{aligned}$$

where $L_1 = -L/2+1$ and $L_2 = L/2$. $\Phi(0, 0)$ is defined as

$$(\phi(L_1, L_1), \dots, \phi(L_1, L_2), \phi(L_1+1, L_1), \dots, \phi(L_1+1, L_2), \dots, \phi(L_2, L_1), \dots, \phi(L_2, L_2))^T.$$

Then for an arbitrary parameter t , we can get the following conclusion from the refinement equation (6),

$$\Phi\left(\frac{t_1+l_1}{m}, \frac{t_2+l_2}{m}\right) = T_{l_1, l_2} \Phi(t_1, t_2) \quad (11)$$

where $l_1, l_2 = 0, 1, \dots, m-1$, and the matrix T_{l_1, l_2} is defined by equation (10).

For an arbitrary parameter $t \in [0, 1]^2$, we decompose it in the m -ary system as follows:

$$t_1 = \sum_{j=1}^{\infty} k_{1j} m^{-j}; \quad t_2 = \sum_{j=1}^{\infty} k_{2j} m^{-j} \quad (12)$$

and sequences $\{k_{1j}\}$ and $\{k_{2j}\}$ can be obtained, where $k_{ij} \in \{0, 1, \dots, m-1\}$.

By defining the operator σ as

$$\sigma t_i = t_i - k_{i1} m^{-1} = \sum_{j=2}^{\infty} k_{ij} m^{-j}, \quad i = 1, 2 \quad (13)$$

we have

$$\Phi(t) = \prod_{i=1}^{\infty} T_{k_{i1}, k_{i2}} \Phi(\sigma t) \quad (14)$$

Using equations (12), (13) and (14) recursively, we get

$$\Phi(t) = \prod_{i=1}^n T_{k_{1,n-i+1}, k_{2,n-i+1}} \Phi(0), n \rightarrow \infty \quad (15)$$

2 Evaluation Algorithm-I

Now we describe the evaluation problem precisely. Given a face F_j on the control quad-mesh and a point in the face F_j with parameters (t_1, t_2) , where $t_1, t_2 \in [0, 1]$, find the value of the subdivision surface $f(t_1, t_2)$ at this point corresponding to the subdivision scheme S .

The limit surface generated by S can be written in terms of the basic limit function as^[3]

$$S^\infty P^0(t) = \sum_j p_j^0 \phi_S(t - j) \quad (16)$$

where $t = (t_1, t_2)^T$, $j \in \mathbb{Z}^2$, and $P^0 = \{p_j^0\}$ is the initial control point set.

By the finite support of S and its symmetric property, we can get the limit surface corresponding to the face F_j ,

$$S^\infty \bar{P}^0(t) = \sum_{-L/2+1}^{L/2} \sum_{-L/2+1}^{L/2} p_j^0 \phi_S(t - j) \quad (17)$$

where $\bar{P}^0 = \{p_{i,j}^0, i, j \in [-L/2+1, L/2]\}$ is the control points vector corresponding to F , and L is defined by the equation (7). The function values $\phi_S(t - j)$, $j \in [-L/2+1, L/2]^2$ can be obtained from equation (15).

Now, we give the evaluation algorithm-I as follows.

Algorithm-I:

Step 1. For the subdivision scheme S satisfying the equation (6), we construct $m \times m$ square matrices $T_{1,i,2}$ via equation (9);

Step 2. Given parameters (t_1, t_2) , where $t_1, t_2 \in [0, 1]$, we decompose them in the m -ary system by equation (12), and get the number sequence $\{k_{1i}, k_{2i}\}_{i=1}^n$, where n represents the given depth;

Step 3. The matrix T is constructed by $T = \prod_{i=1}^n T_{k_{1,n-i+1}, k_{2,n-i+1}}$;

Step 4. The column $L(L-1)/2$ of the matrix T corresponds to the basis function values $\phi_S(t - j)$, $j \in [-L/2+1, L/2]^2$, and we denote

$$A := (T_{1,L(L-1)/2}, \dots, T_{L^2, L(L-1)/2});$$

Step 5. The location of (t_1, t_2) in the limit surface corresponding to S can be obtained, $f(t_1, t_2) = A \bar{P}^0$, where \bar{P}^0 is defined as

$$\bar{P}^0 = (p_{L_1, L_1}^0, p_{L_1+1, L_1+2}^0, \dots, p_{L_2, L_2}^0)^T \quad (18)$$

where $L_1 = -L/2+1$ and $L_2 = L/2$. In the case of ternary quad-mesh subdivision schemes (Li, *et al.*^[12]), Fig.1 illustrates the details of control points \bar{P}^0 corresponding to face F .

3 Evaluation Algorithm-II

In this section, we will deal with regular quad-mesh interpolatory subdivision schemes constructed based on tensor-product such as the schemes proposed by Kobbelt^[8] and Li, *et al.*^[12].

Based on the limit function ϕ_S of a convergent univariate subdivision scheme S , the basic limit function of the related tensor-product scheme $S \times S$ can be constructed as

$$\phi_{S \times S}(t_1, t_2) = \phi_S(t_1) \phi_S(t_2) \quad (19)$$

Then, the limit surface generated by $S \times S$ from the initial control points P_0 is

$$(S \times S)^\infty P^0(t_1, t_2) = \sum_{(i,j) \in \mathbb{Z}^2} P_{i,j}^0 \phi_S(t_1 - i) \phi_S(t_2 - j) \quad (20)$$

Consequently, we give the algorithm-II based on the tensor-product. Firstly we can get the $\{\phi_s(t_1 - i)\}_{-L/2+1}^{L/2}$ and $\{\phi_s(t_2 - j)\}_{-L/2+1}^{L/2}$, then have the $\{\phi_{S \times S}(t_1 - i, t_2 - j) : i, j \in [-L/2 + 1, L/2]\}$ via the equation (19).

The evaluation algorithm-II is given as follows.

Algorithm-II:

Step 1. For the univariate subdivision scheme S satisfying the refinement equation (6), where $m \in Z$. We construct m square matrices T_i via equation

$$T_i = (T_{ij})_{L \times L}, T_{ij} = a_{mi-j+l+L(m-1)/2};$$

Step 2. Given parameters t_1, t_2 where $t_1, t_2 \in [0, 1)$, we decompose them in the m -ary system via equation (12), and get the number sequences $\{k_{1i}\}_{i=1}^n, \{k_{2i}\}_{i=1}^n$, where n represents the given depth;

Step 3. The matrices $T1, T2$ are constructed by $T1 = \prod_{i=1}^n T_{k_{1,n-i+1}}, T2 = \prod_{i=1}^n T_{k_{2,n-i+1}}$;

Step 4. The column $L/2$ of the matrices $T1, T2$ corresponds to the basis function values

$$\phi_s(t_1 - i), i \in [-L/2 + 1, L/2]; \phi_s(t_2 - j), j \in [-L/2 + 1, L/2]$$

respectively, and we denote

$$A_{1,L} := (T1_{1,L/2}, \dots, T1_{L,L/2}), A_{2,L} := (T2_{1,L/2}, \dots, T2_{L,L/2}).$$

Then the weight vector A is defined as

$$A := ((A1^T A2)_{1,1}, (A1^T A2)_{1,2}, \dots, (A1^T A2)_{2,1}, \dots, (A1^T A2)_{L,L}).$$

Step 5. The location of (t_1, t_2) in the limit surface corresponding to $S \times S$ can be obtained, $f(t_1, t_2) = \overline{AP}^0$, where \overline{P}^0 is defined by equation (18).

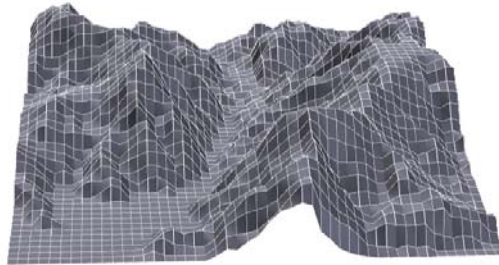


Fig.2 The quad-mesh data for resampling (2500 vertices, 2401 faces)

4 Numerical Examples

By taking quad-mesh ternary interpolatory subdivision^[12] scheme as the subdivision example, we compare the time-consuming performances of the dynamic stencil method^[16] with that of the algorithms proposed in this paper. All the computations have been performed on a Pentium(R) 4 CPU 2.40GHz PC by using VC++6.0 codes based on half-edge data structure. Fig.2 presents a regular quad-mesh on which we implement resampling with different evaluation algorithms.

Table 1 Runtime of algorithm based on dynamic stencil method

Number	Subdivision depth	Runtime (ms)	Errors
100	5	3 625	10^{-3}
	10	7 641	10^{-5}
	15	11 704	10^{-8}

Table 1 gives the runtime results corresponding to the dynamic stencil subdivision evaluation method with different numbers of sampling points, subdivision depths and error estimation (The error here (hereafter) is defined as $\|(t_1, t_2) - (t'_1, t'_2)\|_2$, where (t_1, t_2) denotes the given parameter, $t'_i = \sum_{j=1}^n k_{i,j} m^{-j}, i=1,2$, and n denotes subdivision

depth. For the C^k ($k \geq 1$) subdivision schemes, the error stated above can reflect the accuracy of the algorithms).

The method dynamically constructs the 2-ring of the face F with the similar idea in^[16] which deals with Loop subdivision scheme. Most of operations are dependent on the mesh data structure.

Table 2 shows the runtime results corresponding to algorithm-I with different numbers of sampling points and subdivision depths respectively, where algorithm-I is based on 2D subdivision masks. From Table 2, we can find that algorithm-I is more efficient than the dynamic stencil method for its static processing method independence of mesh data structure.

Table 2 Runtime of algorithm-I based on 2D subdivision masks

Number	Subdivision depth	Runtime (ms)	Errors
1 000	5	406	10^{-3}
	10	922	10^{-5}
	15	1 437	10^{-8}
10 000	5	4 140	10^{-3}
	10	9 297	10^{-5}
	15	14 453	10^{-8}

Table 3 shows the runtime results corresponding to algorithm-II based on tensor-product. In this case, much more sampling points are evaluated. From Table 3, we find that algorithm-II performs even better than algorithm-I, for its small matrices 6×6 with less storage.

Table 3 Runtime of algorithm-II based on tensor-product

Number	Subdivision depth	Runtime (ms)	Errors
10 000	5	31	10^{-3}
	10	78	10^{-5}
	15	109	10^{-8}
100 000	5	422	10^{-3}
	10	812	10^{-5}
	15	1 203	10^{-8}

5 Conclusions

In this paper, we have demonstrated two algorithms for evaluating subdivision surfaces generated by regular quad-mesh interpolatory subdivision schemes. Algorithms proposed in this paper are independent of the mesh data structure, and most of the operations in our algorithms are the calculations of the finite matrix sequences. Therefore, many exhausting works such as computing the neighborhoods are avoided. Our algorithms can be implemented easily with low storage. The numerical results show that they are more efficient compared with the dynamic stencil subdivision evaluation method. Especially, algorithm-II based on tensor-product has more advantages as mentioned in section 5. Algorithm-I can be generalized to non-symmetric or non-tensor-product quad-mesh interpolatory subdivision schemes. However, algorithms proposed in this paper can only be implemented for the regular quad-mesh case.

For the irregular mesh case, the evaluation near extraordinary points can be carried out by applying the subdivision process locally via the dynamic stencil method until the evaluation point has a sufficiently large regular neighborhood.

In the future work, we will consider the question of evaluation derivatives of surfaces generated by interpolatory subdivision schemes and more efficient evaluation algorithms to process arbitrary topology mesh.

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