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基于 Lagrange-Newton 方法的零亏格网格的参数化*

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Spherical Parameterization of Genus-Zero Meshes Using the Lagrange-Newton Method

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Abstract: This paper addresses the problem of spherical parameterization, i.e., mapping a given polygonal surface of genus-zero onto a unit sphere. There exist some methods to deal with the problem in literatures. In this paper, an improved algorithm is constructed for parameterization of genus-zero meshes and aim to obtain high-quality surfaces fitting with PHT-splines. This parameterization consists of minimizing discrete harmonic energy subject to spherical constraints and solving the constrained optimization by the Lagrange-Newton method. Several examples show that parametric surfaces of PHT-splines can be constructed adaptively and efficiently to fit given meshes associated with the parameterization results.

Key words: spherical parameterization; genus-zero mesh; discrete harmonic energy; constrained optimization; Lagrange-Newton method

摘 要: 考虑零亏格网格的球面参数化问题,即将给定的零亏格多边形曲面一一映射到单位球面上.已有一些方法解决该问题.针对应用 PHT 样条进行曲面拟合的需要,对于给定的零亏格网格,提供了一个改进的算法.这种参数化方法主要包含两部分:一是限制在球面约束条件下,极小化离散调和能量;二是使用 Lagrange-Newton 方法求解带约束的优化问题.几个例子演示了参数化的结果,并且说明了应用该参数化结果,能够用 PHT 样条曲面更好地拟合给定网格曲面.

关键词: 球面参数化;零亏格网格;离散调和能量;带约束的优化问题;Lagrange-Newton 方法

1 Introduction

Parameterization is an important problem in meshing data processing. A parameterization of a polygonal mesh in 3D space can be viewed as a one-to-one mapping from the given mesh to a suitable domain which is also a mesh.

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Typically, if the mesh is simple, the suitable domain is a connected region on the plane; and if the mesh is with genus-zero, the domain is a unit sphere. Usually, the meshes consist of triangles. Hence the mappings are piecewise linear and we only need to compute the vertex positions of the triangles.

Parameterizations have many applications in various fields, including texture mapping, surface approximation and remeshing, scattered data fitting, repair of CAD models, morphing, reparameterization of spline surfaces, and so on[1]. For a specified application, some parameterization results might behave better than others. Here the choice of different parameterizations depends heavily on the application details. Possibly a parameterization result behaves better for texture mapping, but worse for surface fitting. For a genus-zero mesh, there have existed some methods to parameterize it onto a sphere. But according to our experiences, these results are unfit for surface fitting with PHT-splines[2]. In Ref.[3], we proposed a method to obtain parameterization results suitable for surface fitting. This method has some disadvantages as well, such as with the local bad shapes of the triangles. Hence in this paper, we introduce a new method to parameterize a genus-zero mesh such that the triangles in the parameterization domain are with better shapes and a surface fitting algorithm in PHT-splines can generate a better result.

1.1 Related works

Now we review some previous works on mesh parameterizations. For a more detailed summary, please refer to Refs.[1,4].

Most of planar parameterization methods establish mappings from a simple mesh to a planar domain by solving linear equations system, such as Refs.[5−7]. Eck, *et al*.[8] introduced the discrete harmonic mapping to parameterize a simple mesh. The methods proposed in the current paper and Ref.[3], which are applied to genus-zero meshes, share the same model as in the Eck's method. The Eck's method is a quadratic minimization problem and also can be reduced to a linear system of equations. But our model is for genus-zero meshes.

Spherical parameterization methods build mappings from a genus-zero surface to a sphere. There have been a lot of interesting and novel methods in spherical parameterization. Many of these methods are very similar to those mapping simple meshes onto planar domains, whereas some of the linear methods become non-linear versions. Haker, *et al*.[9] used a method which maps the given genus-zero mesh into the plane and then uses stereographic projection to map to a sphere. Gu and Yau[10] gave an important point that harmonic maps from a closed genus-zero mesh to a unit sphere are conformal, which means harmonic and conformal maps are the same with genus-zero meshes. Later, they proposed an iterative method which approximates a harmonic map without splitting. Praun and Hoppe[11] extended the definition of stretch to consider a spherical parameterization. For any point inside, the Jacobian map provides a local approximation for the mapping. Consequently, they defined the stretch over the triangle with the singular values of the Jacobian map.

In Ref.[3], a hierarchical method to parameterize a genus-zero mesh to a unit sphere was presented. In the method, a model of spherical parameterization based on minimizing the discrete harmonic energy[8] was proposed. Then a model solving algorithm based on reset PRP conjugate gradient method was introduced. In the algorithm, a hierarchical idea was applied in order to reduce computing time. All points were fixed layer by layer. Experientially, this method is efficient, practical, and versatile for different surfaces. It is more important that according to our experiences, the PHT-spline surface fitting results with hierarchical parameterization results are better than those with other parameterization results. However, in the result meshes, some triangles between two layers might have bad shapes. And in the solving algorithm, there were many user-specified parameters.

1.2 Our contribution

In this paper, we solve the model of mapping a genus-zero surface to the unit sphere in Ref.[3] using the

Lagrange-Newton method. At first, the model of spherical parameterization based on minimizing the discrete harmonic energy is reviewed. Then we use the Lagrange-Newton method to introduce a new stable spherical parameterization algorithm which obtains result meshes with good shapes.

Based on our new parameterization results, a surface fitting algorithm with PHT-splines[2] can generate good results, where parametric surfaces can be constructed efficiently and adaptively to fit genus-zero meshes.

The paper is organized as follows. In Section 2, we review the model and the algorithm proposed in Ref.[3]. In Section 3, the Lagrange-Newton method is adopted to solving the model. Then, the illustrative results of our solving algorithm are provided in Section 4. Finally, we conclude this paper in Section 5.

2 Review of the Model Based on Discrete Harmonic Mappings

In its surface fitting, the parameterization of a genus-zero mesh over some standard domain is needed. Usually the triangles in given meshes are with good shape, i.e., that the three edges of the triangle do not change dramatically in their lengths. In order to obtain good fitting results, we need the triangles in the parameterization domain to be with good shapes as well. Hence, we should propose a mapping from the given mesh to a unit sphere preserving the shapes of the triangles.

Unfortunately, most of the existing spherical parameterization methods do not fulfil this requirement. For a simple mesh, Eck, *et al*.[8] proposed a discrete harmonic method which preserves the aspect ratios of triangles. So in Ref.[3], the Eck's model is generalized to spherical parameterization. In the rest of the section, we review the model and the solving algorithm.

2.1 The model

A triangle mesh $M=(V,E)$ is given with a set of vertices $V=\{v_1,v_2,\ldots,v_n\}$ and a set of edges $E=\{(v_i,v_j)\,|\,v_iv_j$ is an edge of the mesh $M\}$. Suppose that h is any piecewise linear mapping from M to a unit sphere $S^2\subset\mathbb{R}^3$ with the restriction conditions

$$\|h(v_i)\|^2=1,\quad \forall v_i\in V \tag{1}$$

The mapping h is uniquely determined by its value $h(v_i)$ at the vertices of M. Then the discrete harmonic energy of the mapping h associated with the mesh M is defined as

$$f(h,M)=\frac{1}{2}\sum_{(v_i,v_j)\in E}\kappa_{ij}\|h(v_i)-h(v_j)\|^2 \tag{2}$$

where the spring constants κ_{ij} may be computed in many ways. In most cases and the rest of the paper, uniform spring constants are used, i.e., $\kappa_{ij}=1$, for any i and j.

Let $h(v_i)=X_i\in\mathbb{R}^3$; $\boldsymbol{x}=(X_1^T,X_2^T,\ldots,X_n^T)^T\in\mathbb{R}^m$, where $m=3n$; $D(i)=\{j\,|\,(v_i,v_j)\in E\}$; and $d(i)$ denotes the element number of the set $D(i)$. Then we can setup the parameterization model by minimizing the discrete harmonic energy in (2) with spherical constraints:

$$\begin{cases}\min f(\boldsymbol{x})=\dfrac{1}{2}\displaystyle\sum_{i=1}^{n}\sum_{j\in D(i)}\kappa_{ij}\|X_i-X_j\|^2\\ \text{s.t. } c_i(\boldsymbol{x})=\|X_i\|^2-1=0,\quad i=1,\ldots,n\end{cases} \tag{3}$$

where, $\boldsymbol{x}$ is call the vector of optimization variables, $f(\boldsymbol{x})$ the objective function to be minimized, $c(\boldsymbol{x})=(c_1(\boldsymbol{x}),c_2(\boldsymbol{x}),\ldots,c_n(\boldsymbol{x}))^T$ the vector of equality constraints.

2.2 The algorithm

The size of the constrained nonlinear programming problem (3) is quite large and these constrains cannot easily be eliminated. In Ref.[3], a penalty function method is used to solve this optimization problem, which needs

to solve a series of unconstrained optimization problem. The reset PRP conjugate gradient algorithm is used to solving the large-scale unconstrained optimization problem. The convergence rate is very slow. In order to reduce computing time, we use a hierarchical idea. All points are fixed layer by layer. After some points are fixed, the spherical constrains may be ignored, and the positions computation the other points is similar to those in planar parameterization. Finally, the points are mapped onto a unit sphere. This procedure leads to a disadvantage that some triangles along or near the boundaries of two layers do not have good shapes. See Fig.1 and Fig.2 for examples.

On the other hand, the Lagrange-Newton algorithm is suitable to solve the equality constraints optimization problem. The object function and all the constraints of the model (3) are quadratic. We can easily give the explicit expressions of the second order information to be computed in the Lagrange-Newton method. Our main work is to pre-treat data to simplify the process. This is what we do in the next section.

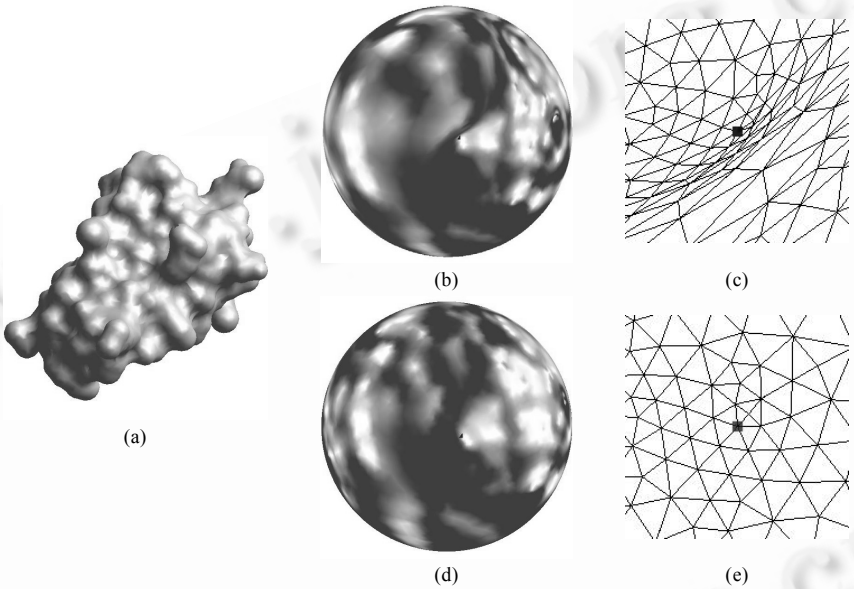

Fig.1 The parameterization results of the model Blob. (a): Original meshes; (b): Parameterization result using the hierarchical method in Ref.[3]; (d): Parameterization result using the Lagrange-Newton method in this paper; (c), (e): Local details comparison of the parameterization results (b) and (d)

图1 模型Blob的参数化结果.(a)为原始网格;(b)为使用文献[3]中分级方法得到的参数化结果; (d)为使用本文中Lagrange-Newton方法得到的参数结果; (c),(e)为参数化结果(b)和(d)的细节比较

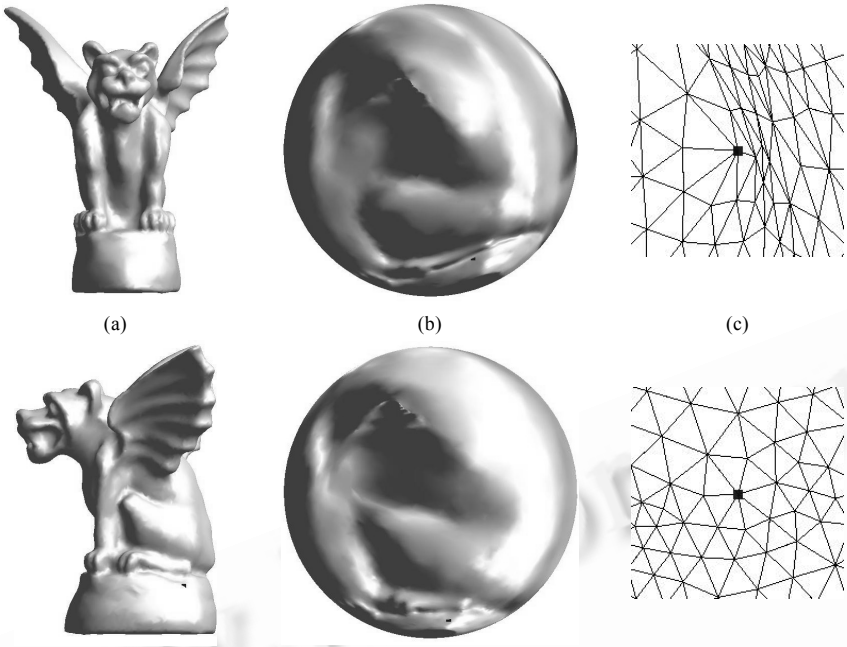

Fig.2 The parameterization results of the model Gargoyle. (a), (d): Different sides of the original mesh; (b): Parameterization result using the hierarchical method in Ref.[3]; (e): Parameterization result using the Lagrange-Newton method in this paper; (c), (f): Local details comparison of the parameterization results (b) and (e)

图2 模型Gargoyle的参数化结果.(a),(d)为原始网格的两个侧面;(b)为使用文献[3]中分级方法得到的参数化结果;(e)为使用本文中Lagrange-Newton方法得到的参数结果;(c),(f)为参数化结果(b)和(e)的细节比较

3 The Lagrange-Newton Method

The Lagrange-Newton method, one of the most efficient numerical methods of solving optimization problems, is developed for problems with equality-type constraints. In the method, the Newton procedure is applied to the first-order optimality system, which has the form of a system of equations. The Lagrange-Newton method is locally quadratically convergent to the solution. This approach has been successfully applied to a class of nonlinear constrained optimization problems.

3.1 Algorithm scheme

At first, we review the general scheme of the Lagrange-Newton method. For an equality-type constrained optimization problem,

$$\begin{cases} \min\limits_{x\in\mathbb{R}^m} f(\boldsymbol{x}) \\ \text{s.t. } c(\boldsymbol{x}) = 0 \end{cases} \tag{4}$$

$\boldsymbol{x}$ is a Kuhn-Tucker point[12] if and only if $\boldsymbol{\lambda}=(\lambda_1,\lambda_2,...,\lambda_n)^T\in\mathbb{R}^n$ exists,

$$\nabla f(\boldsymbol{x})-\nabla c(\boldsymbol{x})^T\boldsymbol{\lambda}=0,\quad -c(\boldsymbol{x})=0 \tag{5}$$

According to the definition of the Lagrange function

$$L(\boldsymbol{x},\boldsymbol{\lambda})=f(\boldsymbol{x})-\boldsymbol{\lambda}^T c(\boldsymbol{x}) \tag{6}$$

formulas (5) virtually solve the stable points of the Lagrange function (6). So all methods based on formulas (5) are

called the Lagrange methods. Given a current iterative point $(\boldsymbol{x}^k \in \mathbb{R}^m, \boldsymbol{\lambda}^k \in \mathbb{R}^n)$, the Newton step of formulas (5) is $(\delta_{\boldsymbol{x}}{}^k, \delta_{\boldsymbol{\lambda}}{}^k)$ which satisfies,

$$\begin{pmatrix} W(\boldsymbol{x}^k, \boldsymbol{\lambda}^k) & -A(\boldsymbol{x}^k) \\ -A^T(\boldsymbol{x}^k) & 0 \end{pmatrix} \begin{pmatrix} \delta_{\boldsymbol{x}}{}^k \\ \delta_{\boldsymbol{\lambda}}{}^k \end{pmatrix} = -\begin{pmatrix} \nabla f(\boldsymbol{x}^k) - A(\boldsymbol{x}^k)\boldsymbol{\lambda}^k \\ -c(\boldsymbol{x}^k) \end{pmatrix} \tag{7}$$

Where,

$$\begin{cases} A(\boldsymbol{x}) = \nabla c(\boldsymbol{x})^T \\ W(\boldsymbol{x}, \boldsymbol{\lambda}) = \nabla^2 f(\boldsymbol{x}) - \sum_{i=1}^{n} \lambda_i \nabla^2 c_i(\boldsymbol{x}) \end{cases} \tag{8}$$

It is well known that the basic Newton iteration only achieves local convergence. A strategy for controlling the step size is required to obtain global convergence. So we define a penalty function,

$$P(\boldsymbol{x}, \boldsymbol{\lambda}) = \| \nabla f(\boldsymbol{x}) - A(\boldsymbol{x})\boldsymbol{\lambda} \|^2 + \| c(\boldsymbol{x}) \|^2 \tag{9}$$

for determining the step size α_k of the k th iteration, because the correction $(\delta_{\boldsymbol{x}}{}^k, \delta_{\boldsymbol{\lambda}}{}^k)$ satisfies

$$\left((\delta_{\boldsymbol{x}}{}^k)^T, (\delta_{\boldsymbol{\lambda}}{}^k)^T \right) \nabla P(\boldsymbol{x}^k, \boldsymbol{\lambda}^k) = -2P(\boldsymbol{x}^k, \boldsymbol{\lambda}^k) \le 0 \tag{10}$$

The direction represents a descent of $P(\boldsymbol{x}, \boldsymbol{\lambda})$ at $(\boldsymbol{x}^k, \boldsymbol{\lambda}^k)$. Then the new iteration is given by

$$\boldsymbol{x}^{k+1} = \boldsymbol{x}^k + \alpha_k \delta_{\boldsymbol{x}}{}^k, \quad \boldsymbol{\lambda}^{k+1} = \boldsymbol{\lambda}^k + \alpha_k \delta_{\boldsymbol{\lambda}}{}^k \tag{11}$$

In the following, the line search Lagrange-Newton algorithm based on formula (7) is given.

Algorithm 1. The Lagrange-Newton Algorithm.

1.1. Given the initial value $(\boldsymbol{x}^0, \boldsymbol{\lambda}^0)$, the tolerance error $\varepsilon \ge 0$, and $\beta = 0.5$. Let $k := 0$;

1.2. Compute $P(\boldsymbol{x}^k, \boldsymbol{\lambda}^k)$. If $P(\boldsymbol{x}^k, \boldsymbol{\lambda}^k) \le \varepsilon$, stop the computation; Otherwise, solve formula (7) to obtain $(\delta_{\boldsymbol{x}}{}^k, \delta_{\boldsymbol{\lambda}}{}^k)$, and let $\alpha \leftarrow 1$.

1.3. If

$$P(\boldsymbol{x}^k + \alpha\delta_{\boldsymbol{x}}{}^k, \boldsymbol{\lambda}^k + \alpha\delta_{\boldsymbol{\lambda}}{}^k) \le (1 - \beta\alpha) P(\boldsymbol{x}^k, \boldsymbol{\lambda}^k) \tag{12}$$

Go to Step 1.4; Otherwise, obtain $(\delta_{\boldsymbol{x}}{}^k, \delta_{\boldsymbol{\lambda}}{}^k)$, and return to Step 1.3.

1.4. Update

$\boldsymbol{x}^{k+1} = \boldsymbol{x}^k + \alpha_k \delta_{\boldsymbol{x}}{}^k$, $\boldsymbol{\lambda}^{k+1} = \boldsymbol{\lambda}^k + \alpha_k \delta_{\boldsymbol{\lambda}}{}^k$.

Set $k \leftarrow k + 1$, and return to Step 1.2.

3.2 Matrix computation of the model

For the given optimization model (3) of spherical parameterization, we can get the expression of all matrices *n* the Lagrange-Newton method.

Denote the sub-matrices *W* and *A* of the Hessian matrix n the formula (7) as the form of block matrices,

$$\begin{pmatrix} W_{11} & W_{12} & \cdots & W_{1n} \\ W_{21} & W_{22} & \cdots & W_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ W_{n1} & W_{n2} & \cdots & W_{nn} \end{pmatrix}, \quad \begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1n} \\ A_{21} & A_{22} & \cdots & A_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ A_{n1} & A_{n2} & \cdots & A_{nn} \end{pmatrix},$$

where, $$W_{ij} = \frac{\partial^2 L}{\partial X_i \partial X_j} = \begin{pmatrix} \frac{\partial^2 L}{\partial X_{i1} \partial X_{j1}} & \frac{\partial^2 L}{\partial X_{i1} \partial X_{j2}} & \frac{\partial^2 L}{\partial X_{i1} \partial X_{j3}} \\ \frac{\partial^2 L}{\partial X_{i2} \partial X_{j1}} & \frac{\partial^2 L}{\partial X_{i2} \partial X_{j2}} & \frac{\partial^2 L}{\partial X_{i2} \partial X_{j3}} \\ \frac{\partial^2 L}{\partial X_{i3} \partial X_{j1}} & \frac{\partial^2 L}{\partial X_{i3} \partial X_{j2}} & \frac{\partial^2 L}{\partial X_{i3} \partial X_{j3}} \end{pmatrix}, \quad A_{ij} = \frac{\partial^2 L}{\partial \lambda_i \partial X_j} = \begin{pmatrix} \frac{\partial^2 L}{\partial \lambda_i \partial X_{j1}} \\ \frac{\partial^2 L}{\partial \lambda_i \partial X_{j2}} \\ \frac{\partial^2 L}{\partial \lambda_i \partial X_{j3}} \end{pmatrix}.$$

Here W_{ij} is a 3×3 matrix, and A_{ij} is a 3×1 vector. Usually, the uniform spring constants $\kappa_{ij}=1$ are used. From the model (3), we can obtain the Lagrange function as follows:

$$L(\boldsymbol{x},\boldsymbol{\lambda})=f(\boldsymbol{x})-\sum_{i=1}^{n}\lambda_i c_i(\boldsymbol{x})=\frac{1}{2}\sum_{i=1}^{n}\sum_{j\in D(i)}\|X_i-X_j\|^2-\sum_{i=1}^{n}\lambda_i(\|X_i\|^2-1).$$

And, in term of the formula (8), all the matrices to be computed can be expressed as follows:

$$W_{ij}=\begin{cases}(2d(i)-2\lambda_i)I_3, & i=j\\ -2I_3, & i\neq j, j\in D(i),\\ 0, & \text{otherwise}\end{cases}\qquad A_{ij}=\begin{cases}2X_i, & i=j\\ 0, & \text{otherwise}\end{cases}.$$

The right part of equation (7) is

$$\nabla_{X_i}f(\boldsymbol{x})=2\sum_{i\in D(i)}(X_i-X_j).$$

Now, we have obtained the expression of all the matrices. In the following subsection, how to solve the corresponding large-scale linear system (7) is discussed.

3.3 Solving the linear system

Denote the linear system (7) as

$$Hx=b \tag{13}$$

It is easy to show that the matrix H is highly sparse and symmetrical. In our experience, the case that the matrix H is singular occurs extremely rarely, and we can change the initial value to avoid the singular matrix. Hence, we can assume that the matrix H is nonsingular.

For a large-scale sparse linear system of equations, if its coefficient matrix is symmetric and positive definite, the Cholesky decomposition is generally applied. This method is efficient and robust. The linear system (13) is equivalent to

$$H^THx=H^Tb.$$

According to the properties of the matrix H, the new coefficient matrix H^TH is highly sparse, symmetrical, and positive definite. Then the Cholesky decomposition can be applied onto H^TH. In the implementation, we use a scheme called row-indexed sparse storage mode, which requires storage of only about two times the number of nonzero matrix elements, to store the large coefficient matrix. The row-indexed storage mode is introduced in detail in Ref.[13].

In a valid parameterization result, overlapped parts should be eliminated. We use the same method as in Ref.[3] to detect overlapping. An overlapping is a region on the parameterization result, where there exists reverse triangles, whose normals are opposite to the normals of their neighbor triangles. We find these reverse triangles by testing the orientation of the sequence vertices along the boundary of each face. It is important that the three vertices are recorded in a clockwise turn. This can be computed by estimating the sign of $\left((v_{j_2}-v_{j_1})\times(v_{j_3}-v_{j_1})\bullet v_{j_1}\right)$, where $t_j=(v_{j_1}v_{j_2}v_{j_3})$ is a triangle.

Starting with the given mesh as the initial value, the new mesh can be obtained using Algorithm 1. While overlaps exist in the current mesh, we update the tolerance error by multiplying the error by a factor $\theta\in(0,1)$, and use Algorithm 1 to get a new mesh. This factor may be different for various models. Our experiments showed that the factor θ should be less than 0.5 to decrease the times of detecting overlapping and greater than 0.02 to reduce the runtime between two overlapping detection. Typically, the factor θ is taken 0.1. Repeat this work until there is no overlapped part in the obtained mesh. This mesh is a valid parameterization result.

4 Results and Discussion

The statistical data of some spherical parameterization results using the solving algorithm are provided in Table 1.

From Table 1, it is known that, compared with hierarchical method in Ref.[3], our new method cannot reduce computing time effectively. But the new method is improved at two main points. Firstly, the triangles in new parameterization results are with better shapes. The local comparison will be provided in the next subsection. Secondly, the new parameterization algorithm is more automatically. In the hierarchical algorithm, there are many parameters to be specified by users. And in the new algorithm, only one parameter, which is the global tolerance error, need to be given.

Table 1 Runtime comparison among genus-zero examples

Model	Number of vertices	Num. faces	Runtime (s)	Number of iterations
Bishop	250	496	0.891	16
Blob	8 036	16 068	85	10
Venus face	8 268	16 532	34	3
Venus body	11 362	22 720	52	4
Gargoyle	10 002	20 000	50	4
Cow	11 610	23 216	201	14
Skull	20 002	40 000	560	24

4.1 Local shape comparison

In Figures 1 and 2, the triangle local shape comparison is provided. In all the figures, there is a point labeled with a red square to identify the same vertex in the original meshes, the whole parameterization results with the methods in Ref.[3] and in the current paper, and the local zoom-in triangles, respectively.

With the local zoom-in viewports, we can see that the results using hierarchical method usually have some lathy triangles, and the corresponding triangles in the new results are more better.

4.2 Surface fitting

Based on PHT-spline spaces, parametric surfaces can be constructed efficiently and adaptively to fit a genus-zero mesh after its spherical parameterization has been obtained. Using the PHT-splines proposed in Ref.[2], a surface fitting algorithm is proposed. Now we apply this algorithm to fit genus-zero meshes based on their spherical parameterizations. In Fig.3, we can see that the new result, especially parts around eyes, nose, forehead and cheek, is better than that with the hierarchical parameterization results. Fig.3(a) and Fig.3(d) are the whole fitting surfaces based on the different parameterization results; Fig.3(b) and Fig.3(c) are the local details of Fig.3(a), while Fig.3(e) and Fig.3(f) are the local details of the new result Fig.3(d).

5 Conclusions

We have presented a new approach based on the Lagrange-Newton method to solve the spherical parameterization model of a genus-zero mesh proposed in Ref.[3]. Compared with the hierarchical algorithm, the new approach has improved the results in two points: the local shapes of the triangles and less user-specified parameters. Specially, the PHT-spline surface fitting results with the new parameterization results are better than those with hierarchical results.

In the future, we will focus on the following works:

1. Now, we use the discrete harmonic energy as the objective function, and obtain the results with good shapes. But, the sizes of triangles are different widely. So a better objective function or a new storage

structure may be given.

2. In this paper, the genus-zero surfaces can be parameterized efficiently. The parameterization method of higher genus surfaces can be generalized from this method.

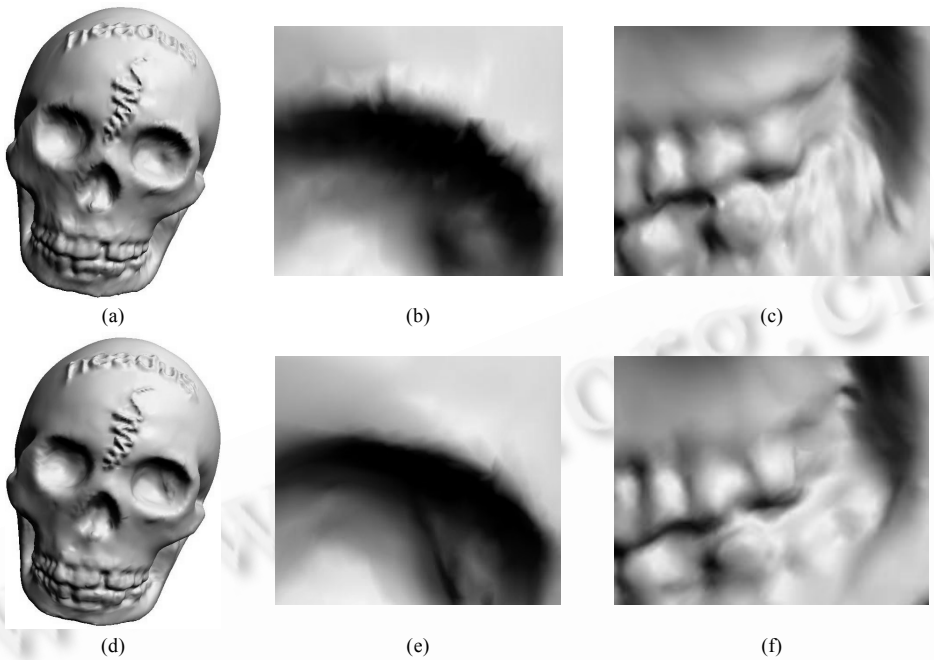

Fig.3 Fitting genus-zero meshes. (a), (d): Whole fitting surfaces based on the parameterization results using the hierarchical method in Ref.[3] and the Lagrange-Newton method in this paper; (b), (e): Local details on the left eye of the model Skull; (c), (f): Local details on the teeth

图3 拟合零亏格网格.(a),(d)为使用两种不同参数化结果(文献[3]中分级方法和本文中的Lagrange-Newton方法)得到的拟合曲面;(b),(e)为模型Skull左眼处的拟合细节;(c),(f)为模型牙齿处的拟合细节

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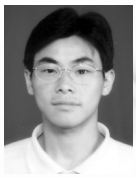

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