NKI 中的本体、框架和逻辑理论*

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Ontologies, Frames and Logical Theories in NKI

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Abstract: NKI (The National Knowledge Infrastructure) is a large-scale knowledge base, which uses frames to represent concepts in ontologies, and uses Horn logic programs as the automated reasoning. The formalizations of ontologies, frames and logical theories in NKI, and the transformations between the formalizations are given, and proved to be functors between ontologies, frames and logical theories if they are taken as categories in the theory of category. The result proved in this paper guarantees that in NKI, the inference based on the Horn logic programs is correct with respect to the knowledge base represented by ontologies and frames.

Key words: ontologies; frames; theories; transformation

摘 要: NKI(国家知识基础设施)是一个大规模知识库,它用框架来表示本体中的概念,用 Horn 逻辑程序作为自动推理.给出 NKI 中的本体、框架和逻辑理论的形式表示以及形式表示之间的转换,并证明如果将本体、框架和逻辑理论看作是 3 个范畴,则这些转换是这 3 个范畴之间的函子.这个结果保证了在 NKI 中,基于 Horn 逻辑程序的推理关于用本体和框架表示的知识库是正确的.

关键词: 本体;框架;理论;转换

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1 Introduction

There are several methods for representing knowledge which have the same expressiveness: classically the first order theories[1,2], frames[3], description logics[4], and conceptual graphs[5]. Every representation has its own presuppositions. When transforming one representation into another to prove that two representations have the same expressiveness, we find that it is necessary to articulate these presuppositions. Usually we prove that if a representation can be faithfully and fully transformed into the first order theories; and if every first order theory can be represented faithfully and fully in the representation, then the representation has the same expressiveness as the first order logic. Another reason to transform a representation into first order theories is given by Horty[6], when he considered the defeasible inheritance networks and the non-monotonity of defeasible inheritance reasoning. Horty[6] proposed that the defeasible inheritance networks

> ...were supplied only with a 'procedural' semantics, according to which the meaning of the representations was supposed to be specified implicitly by the inference algorithms operating on them. It was soon realized, however, that these algorithms could lead to bizarre and unintuitive results in complicated cases, and researchers felt the need to provide an implementation-independent according of the meaning of these network formalisms. One natural idea involved providing a logical interpretation of the networks--interpreting the individual links in the network as logical formulas, and so the entire network as a collection of formulas, whose meaning could then be specified by the appropriate logic...

Ontologies are used in knowledge engineering to share knowledge among different knowledge bases. Hence, ontologies should be neutral to the representation of knowledge. But ontologies should be represented in some formal or informal languages, and languages in any form are not sharable. Therefore, there is a trade-off between the neutrality and the usage of languages.

We assume that (1) ontologies consist of propositions; (2) propositions are not sentences, but are what sentences express: different sentences in different languages (or possibly the same language) can express the same proposition; (3) propositions can be equivalent without being identical. Propositions and ontologies are objects we can talk about, and the content of an ontology consists of the propositions involving concepts in the ontology that are entailed by the constituent propositions of the ontology; and ontologies are comparable in terms of their contents.

NKI[7,8] (The National Knowledge Infrastructure) is a large-scale knowledge base, which uses frames to represent concepts in ontologies, and uses Horn logic programs as the automated reasoning. Hence, NKI includes three classes of "concepts": the first one is the class of concepts in ontologies, the second one is the class of frames which are used to represent concepts in an ontology; and the last one is the class of logical theories which have the equivalent meanings as the concepts and the frames. We formally articulate the presuppositions we make about the frames and ontologies, and show under the presuppositions, the equivalences between three classes to ensure that any query to NKI is given a correct answer.

First we represent an ontology $\mathbf{O}$ as a set $\boldsymbol{F}$ of frames; and then represent the set $\boldsymbol{F}$ of frames as a set $\boldsymbol{T}$ of logical theories. We define the transformations $\sigma:\mathbf{O}\rightarrow\boldsymbol{F}$ and $\tau:\boldsymbol{F}\rightarrow\boldsymbol{T}$ such that σ, τ are congruences, that is, they preserves the partial orders and basic defined operations in $\mathbf{O}$, $\boldsymbol{F}$ and $\boldsymbol{T}$. If $\mathbf{O}$, $\boldsymbol{F}$ and $\boldsymbol{T}$ are taken as the categories in the category theory, then σ and τ are the functors between the categories.

The paper is organized as follows: the next section gives the formal definitions of ontologies, concepts, frames and logical theories, and the comparisons between concepts, frames and logical theories; the third section defines the transformations from an ontology to frames and from frames to logical theories, and proves that the transformations are faithful and structural-property-preserving; the fourth section takes the block world as an example to show how to build an ontology, define frames and transform frames into logical theories in knowledge engineering; the last section concludes the paper.

2 The Concepts, Frames and Logical Theories

2.1 The ontology

An ontology **O** is a quadruple $(O, A, R, \sqsubseteq)$, where

- O is a set of concepts. We assume that $O=U\cup C$ and $U\cap C=\varnothing$, where U is a set of individuals, and C is a class of concepts;
- A is a set of attributes. For every $a\in A$, there is a domain D_a such that a is taken as a function from U to D_a. We use $a(x)=v$ to denote that individual x has value v at attribute a;
- R is a set of relations. Without loss of generality, we assume that every relation is binary on U, that is, for every $r\in R$, $r\subseteq U\times U$, and

  $\mathrm{dom}(r)=\{x\in U\mid \exists y((x,y)\in r)\}$;

  $\mathrm{range}(r)=\{y\in U\mid \exists x((x,y)\in r)\}$;
- $\sqsubseteq$ is a binary relation on O, called the subsumption relation or is a relation. We assume that (1) $\sqsubseteq$ is a partial order on O; (2) for any $x, y\in U$, $x\not\sqsubseteq y$; and (3) for any $\alpha, \beta\in C$, if $\alpha\sqsubseteq\beta$ then for any $x\in U$, $x\sqsubseteq\alpha$ implies $x\sqsubseteq\beta$,

and **O** satisfies the following condition:

(2.1) For any $\alpha, \beta\in C$, if $\alpha\neq\beta$ then there are $x, y\in U$ and an attribute $a\in A$ or a relation $r\in R$ such that either $a(x)\in^{i}\{a(x')\mid x'\sqsubseteq\alpha\}$, $a(x)\in^{1-i}\{a(x')\mid x'\sqsubseteq\beta\}$; or $x\in^{i}\mathrm{dom}^{\alpha}(r)\cup\mathrm{range}^{\alpha}(r)$, $x\in^{1-i}\mathrm{dom}^{\beta}(r)\cup\mathrm{range}^{\beta}(r)$,

where $i=0, 1$; $\in^{0}=\notin$, $\in^{1}=\in$, and

$\mathrm{dom}^{\alpha}(r)=\{x'\sqsubseteq\alpha\mid \exists y'((x', y')\in r)\}$,

$\mathrm{range}^{\alpha}(r)=\{y'\sqsubseteq\alpha\mid \exists x'((x', y')\in r)\}$.

To simplify the notation, we use $x{:}\,\alpha$ to denote that $x\sqsubseteq\alpha$ if $x\in U$ and $\alpha\in C$. We assume the Leibniz law, that is, for any $x, y\in U$, $x\neq y$ if and only if there is an attribute $a\in A$ or a relation $r\in R$ such that $a(x)\neq a(y)$; or for some z, $(x, z)\in r$ and $(y,z)\notin r$; or for some z, $(x,z)\notin r$ and $(y,z)\in r$.

We can define the operations $\sqcap, \sqcup$ on **O**, where **O** is taken as a partial ordered structure.

Definition 2.1. Given two $\alpha, \beta\in O$, the greatest lower bound of α and β, denoted by $\alpha\sqcap\beta$, is a concept $\gamma\in O$ such that $\gamma\sqsubseteq\alpha, \beta$; and for any $\delta\in O$, if $\delta\sqsubseteq\alpha, \beta$ then $\delta\sqsubseteq\gamma$ or $\delta=\gamma$.

The least upper bound of α and β, denoted by $\alpha\sqcup\beta$, is a concept $\gamma\in O$ such that $\gamma\sqsupseteq\alpha, \beta$; and for any $\delta\in O$, if $\delta\sqsupseteq\alpha, \beta$ then $\gamma\sqsubseteq\delta$ or $\delta=\gamma$.

2.2 The frames

Assume that there is a set O of concepts, a set A of attributes and a set R of binary relations.

A string φ of symbols is a statement if either

$$\varphi=[a{:}v],$$

where $a\in A$ and $v\subseteq D_a$; or

$$\varphi=[r{:}u],$$

where $r\in R$ and $u\subseteq U$.

A frame F is a set of statements such that for any $a\in A$, there is at most one $v\subseteq D_a$ such that $[a{:}v]\in F$; and for any $r\in R$, there is at most one $u\subseteq range(r)$ such that $[r{:}u]\in F$. Let $\boldsymbol{F}$ be the set of all the frames.

We define a binary relation $\leq$ between frames, called the inheritance relation: given two frames F and F', we say that F is a sub-frame of F', or F inherits F', denoted by $F\leq F'$, if for any $[a{:}v']\in F'$ there is a v such that $[a{:}v]\in F$ and $v\subseteq v'$; and for any $[r{:}u']\in F'$ there is one u such that $[r{:}u]\in F$ and $u\subseteq u'$.

Corresponding to the Leibniz law, we assume that for any two frames F and F', if $F\neq F'$ and $F\leq F'$ then there is at least one attribute $a\in A$ or one relation $r\in R$ such that either $[a{:}v]\in F$, $[a{:}v']\in F'$ and $v\subset v'$; or $[r{:}u]\in F$, $[r{:}u']\in F'$ and $u\subset u'$.

It is easily proved that $\leq$ is a partial order on $\mathcal{F}$. We define the operations on $\mathcal{F}$.

Definition 2.2. Given two frames F and F', we define two frames, denoted by $F\wedge F'$ and $F\vee F'$ as follows:

- for any $a\in A$, $[a{:}v]\in F\vee F'$ for some $v\subseteq D_a$ if and only if $[a{:}v']\in F$ for some v'; $[a{:}v'']\in F'$ for some v'', and $v=v'\cup v''$;
- for any $r\in R$, $[r{:}u]\in F\vee F'$ for some $u\subseteq U$ if and only if $[r{:}u']\in F$ for some u'; $[r{:}u'']\in F'$ for some u'', and $u=u'\cup u''$;
- for any $a\in A$, $[a{:}v]\in F\wedge F'$ for some $v\subseteq D_a$ if and only if $v=v'\cap v''$ if $[a{:}v']\in F$ for some v' and $[a{:}v'']\in F'$ for some v''; $v=v'$ if $[a{:}v']\in F$ for some v' and $[a{:}v'']\in F'$ for no v''; $v=v''$ if $[a{:}v']\in F$ for no v' and $[a{:}v'']\in F'$ for some v'';
- for any $r\in R$, $[r{:}u]\in F\wedge F'$ for some $u\subseteq U$ if and only if $u=u'\cap u''$ if $[r{:}u']\in F$ for some u' and $[r{:}u'']\in F'$ for some u''; $u=u'$ if $[r{:}u']\in F$ for some u' and $[r{:}u'']\in F'$ for no u''; $u=u''$ if $[r{:}u']\in F$ for no u' and $[r{:}u'']\in F'$ for some u''.

Proposition 2.3. For any $F, F'\in\mathcal{F}$, $F\wedge F'$ and $F\vee F'$ are the greatest lower bound and the least upper bound of F and F' under $\leq$, respectively.

Given a class $\mathcal{F}''$ of frames, any two frames F and F' may not have $F\wedge F'$ and $F\vee F'$ in $\mathcal{F}''$. Hence, we can define $\wedge_{\mathcal{F}''}$ and $\vee_{\mathcal{F}''}$ in $\mathcal{F}''$ as follows:

Definition 2.4. Given two frames F and F', we define $F\wedge_{\mathcal{F}''}F'$ and $F\vee_{\mathcal{F}'}F'$ to be the greatest lower bound and the least upper bound of F and F' in $\mathcal{F}''$, respectively.

Proposition 2.5. Given a class $\mathcal{F}''$ of frames and two frames F, F',

$$F\wedge_{\mathcal{F}''}F'\leq F\wedge F';$$
$$F\vee F'\leq F\vee_{\mathcal{F}''}F'.$$

2.3 The logical theories

Let L be a logical language containing the following symbols: (1) a set U of constants, (2) a set C of unary predicates, (3) a set A of unary functions, and (4) a set R of binary relations, such that for every $\alpha\in C$, $\alpha\subseteq U$; for every $a\in A$, there is a domain D_a and $a: U\to D_a$; and for every $r\in R$, $r\subseteq U\times U$.

A string t of symbols is a term in L if

$$t = x|c|v|a(t'),$$

where x is a variable ; $c\in U\cup C$; for some $a\in A$, $v\in D_a$; $t'\notin D_a$.

A string φ of symbols is a formula in L if

$$\varphi = x{:}\ \alpha|\ t{:}\ \alpha|\ t=s|\ \psi_1\wedge\psi_2|\ \psi_1\to\psi_2|\ \forall x\psi(x),$$

where x is a variable, t, s are terms, $\alpha\in C$.

A formula φ is called a sentence if there is no free variable in φ. A theory T on L is a consistent set of sentences. Let $\mathcal{T}$ be the set of all the theories in L.

Definition 2.6. Given two theories T and T', we say that T' is implied logically by T, denoted by $T\vdash T'$, if for every sentence $\varphi\in T'$, $T\vdash\varphi$, where $\vdash$ is the first order inference. T and T' are equivalent, denoted by $T\equiv T'$, if $T\vdash T'$ and $T'\vdash T$.

Then, $\vdash$ is a partial order on $\mathbf{T}$.

Definition 2.7. Given two theories T and T' in L, we define two theories $T\cup T'$ and $T\cap T'$ as follows:

$$T\cup T'=\mathrm{Cn}(T)\cup \mathrm{Cn}(T');$$
$$T\cap T'=\mathrm{Cn}(T)\cap \mathrm{Cn}(T'),$$

where Cn(T) is the logical closure of T, that is, $\mathrm{Cn}(T)=\{\varphi|T\vdash\varphi\}$.

Proposition 2.8. For any theories T and T', $T\cup T'$ and $T\cap T'$ are the least upper bound and the greatest lower bound of T and T', respectively.

There are two special theories in $\mathbf{T}$: $T_\top$ and $T_\perp$, where $T_\top$ is the inconsistent theory, and $T_\perp$ is the first order theory, that is, $T_\perp=\{\varphi|\vdash\varphi\}$, the set of first order theorems. Then, for any logical theory T, $T_\top\vdash T\vdash T_\perp$.

3 The Transformations

In this section we give the transformations from the ontology to frames and from frames to logical theories such that the transformations preserve the structural properties of three structures: the ontology (i.e., concepts under $\sqsubset$), the frames under the inheritance, and the logical theories under the logical implication.

Proposition 3.1. Given an ontology $\mathbf{O}$, there is a transformation σ: $\mathbf{O}\to\mathbf{F}$ which is a congruence between $\mathbf{O}$ and $\mathbf{F}$, that is, for any $\alpha,\beta\in O$, $\alpha\sqsubset\beta$ if and only if $\sigma(\alpha)\leq\sigma(\beta)$; and

$$\sigma(\alpha\sqcup\beta)=\sigma(\alpha)\vee_{\sigma(\mathbf{O})}\sigma(\beta);$$
$$\sigma(\alpha\sqcap\beta)=\sigma(\alpha)\wedge_{\sigma(\mathbf{O})}\sigma(\beta),$$

where $\sigma(\mathbf{O})=\{\sigma(\alpha)|\alpha\in O\}$.

Proof. Given a concept $\alpha\in O$, define

$$\sigma(\alpha)=\{[a:v]|a\in A\}\cup\{[r:u]|r\in R\},$$

where for any $a\in A$,

$$v=\{a(x)|x\in U,x\sqsubset\alpha\};$$

and for any $r\in R$,

$$u=\{y|\exists x\in U(x\sqsubset\alpha\wedge(x,y)\in r)\}.$$

By (2.1) and the Leibniz law, σ is injective, that is, for any $\alpha,\beta\in O$, if $\alpha\neq\beta$ then $\sigma(\alpha)\neq\sigma(\beta)$.

It is a routine to verify that σ is a congruence. □

Proposition 3.2. Let $\sigma(\mathbf{O})$ be the set of the frames defined in Proposition 3.1. There is a transformation τ: $\sigma(\mathbf{O})\to\mathbf{T}$ which is a congruence between $\sigma(\mathbf{O})$ and $\mathbf{T}$, that is, for any $F, F'\in\sigma(\mathbf{O})$, $F\leq F'$ if and only if $\tau(F)\vdash\tau(F')/\theta$, and

$$\tau(F\sqcup F')=\tau(F)\vee\tau(F');$$
$$\tau(F\sqcap F')=\tau(F)\wedge\tau(F'),$$

where θ is a substitution such that if F and F' are the frames for concepts α and β, respectively, then $\theta=\beta/\alpha$.

Proof. Given a frame $F\in\mathbf{F}$ for some concept $\alpha\in O$, define

$$\tau(F)=\{\forall x{:}\alpha(a(x)\in v)|[a:v]\in F\}\cup\{\forall x{:}\alpha\exists y\in u((x,y)\in r)|[r:u]\in F\}.$$

By (2.1) and the corresponding Leibniz's law, τ is injective, that is, for any $F, F'\in\sigma(\mathbf{O})$, if $F\neq F'$ then $\tau(F)\neq\tau(F')$, that is, either $\tau(F)\not\vdash\tau(F')\theta$ or $\tau(F')\not\vdash\tau(F)\theta'$, where $\theta'=\alpha/\beta$.

Given two frames F, F' for concepts α,β, respectively, such that $\alpha\sqsubset\beta$, assume that $[a:v]\in F$ and $[a:v']\in F'$. Then, $\forall x{:}\alpha(a(x)\in v)$, say $=\varphi$, is in $\tau(F)$ and $\forall x{:}\beta(a(x)\in v')$, say $=\psi$, is in $\tau(F')$. Then, $\varphi\vdash\psi\theta$. By the proof of the above proposition, $v\subseteq v'$. Hence,

$$\forall x{:}\alpha(a(x)\in v)\vdash\forall x{:}\alpha(a(x)\in v');$$
$$\forall x{:}\beta(a(x)\in v')\theta=\forall x{:}\alpha(a(x)\in v');$$
$$\varphi\vdash\psi\theta.$$ □

Classically, the intent of a concept α is a set of properties satisfied by every instance of α. Hence, given two concepts α, β, if $\alpha \sqsubset \beta$, let $I(\alpha)$ be the intent of α, then $I(\alpha)$ should logically imply $I(\beta)$. The transformation τ given in the last proposition says that $I(\alpha) \not\vdash I(\beta)$, but $I(\alpha) \vdash I(\beta)\theta$, here, $I(\alpha)=\tau(\sigma(\alpha))$.

Given a concept α, we use F_α to denote $\sigma(\alpha)$, and use T_α to denote $\tau(F_\alpha)$. We can take $\mathbf{O}$, $\mathbf{F}$ and $\mathbf{T}$ as three categories such that

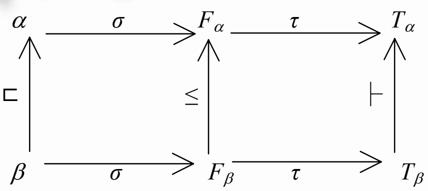
$$\mathbf{O}=(o(\mathbf{O}),\mathrm{hom}_{\mathbf{O}});$$
$$\mathbf{F}=(o(\mathbf{F}),\mathrm{hom}_{\mathbf{F}});$$
$$\mathbf{T}=(o(\mathbf{T}),\mathrm{hom}_{\mathbf{T}}),$$

where o($\mathbf{O}$) is the set of objects in the category, and for any α, $\beta \in$ o($\mathbf{O}$), $\mathrm{hom}_{\mathbf{O}}(\alpha,\beta)$ is a set of morphisms from α to β, namely,

$$o(\mathbf{O})=O;$$
$$\mathrm{hom}_{\mathbf{O}}(\alpha,\beta)=\begin{cases}\{\sqsubset\}, & \text{if } \alpha\sqsubset\beta\\ \Phi, & \text{otherwise,}\end{cases}$$

and similarly we can define $o(\mathbf{F})$, $\mathrm{hom}_{\mathbf{F}}$ and $o(\mathbf{T})$, $\mathrm{hom}_{\mathbf{T}}$.

Then, σ and τ are functors. We have the following commutative diagram:



This diagram guarantees that an ontology, its representation and its reasoning are coincident.

4 An Example

In this section we use the commonly-used block world as an example to show a simple ontology, its frame and logical representations.

Assume that there are five blocks {a, b, c, d, e} and a desk {D}, and every block has length v_0, width v_1 and height v_2. Let M = {a, b, c, d, e, D}.

Let $\mathbf{O}$ be the ontology to describe the blocks. Then,

O = {block, desk, state, a, b, c, d, e, D};

A = {*height*, *width*, *length*};

R = {above},

where

C = {block, desk, state};

U = {a, b, c, d, e, D},

and a state is a structure **s** = (M, {above, top}), where

top $\subseteq M$;

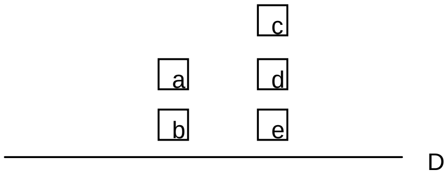
above $\subseteq M \times M$,

and the unary predicate top is defined such that for any $x \in U$,

$$\mathrm{top}(x) \Leftrightarrow \neg\exists y\ (\mathrm{above}(y, x)).$$

The state **s** given in the following diagram is described by such a structure **s** = (M, {above, top}), where top = {a, c};

above = {(a, b), (b, D), (c, d), (d, e), (e, D)},



We assume that the ontology **O** includes the following set of axioms:

$$T=\{\forall x,y,z\,(\text{above}(x,y)\wedge\text{above}(y,z)\rightarrow\text{above}(x,z)),$$
$$\forall x\,(\neg\,\text{above}(\text{D},x)),$$
$$\forall x\,(\text{top}(x)\leftrightarrow\forall y\,(\neg\,\text{above}(y,x)))\}.$$

A move is an action, taken as a mapping from states to states, where the mapping satisfies the following condition: for any states $\mathbf{s}_1$ and $\mathbf{s}_2$, let $\mathbf{s}_1=(M,\{\text{top}_1,\text{above}_1\})$ and $\mathbf{s}_2=(M,\{\text{top}_2,\text{above}_2\})$, then

$$|\,\Delta(\text{top}_1,\text{top}_2)\,|=2,$$

where Δ is the symmetric difference of sets.

We define a frame for block as follows:

deframe : block
{
is-a : object
instances : a, b, c, d, e
attribute : $length=v_0$
attribute : $width=v_1$
attribute : $height=v_2$
relation : above
}

and a frame for block a:

deframe : a
{
is-a : block
attribute : $length=v_0$
attribute : $width=v_1$
attribute : $height=v_2$
}

and a frame for block a in state **s**:

deframe : a in **s**
{
is-a : block
attribute : $length=v_0$
attribute : $width=v_1$
attribute : $height=v_2$
relation : above(a, b), above(a, D)
}

We have three theories for block, a and a in **s**:

$T_{\text{block}}=\{$ $\forall x$: block(x : object),
$\forall x$: block($length(x)=v_0$),
$\forall x$: block($width(x)=v_1$),
$\forall x$: block($height(x)=v_2$)};

$T_{\text{a}}=\{$ a : block,
$length(\text{a})=v_0$,
$width(\text{a})=v_1$,
$height(\text{a})=v_2$};

$T_{\text{a in }\mathbf{s}}=\{$ a : block,

$length$(a) = v_0,
$width$(a) = v_1,
$height$(a) = v_2,
above(a, b),
above(a, D)}.

The descriptions given above seem too simple, because they are just the descriptions of objects. In fact, the descriptions become much complicated when we consider processes, where we should use the types to distinguish the objects, processes, actions, states, etc.

The frame for state is given as follows:

deframe : state
{
is-a : process
haspart : *universe M*
haspart : *relation* : top
haspart : *relation* : above
axiom: $\forall x \in M\,(x{:}\,\sigma \vee x : \tau)$;
top$\subseteq M$;
above$\subseteq M \times M$;
$\forall x \in M\,(x{:}\,\tau \rightarrow x \in$top);
$\forall x \in M\,(x \in$top $\rightarrow \neg\exists y \in M\,((y, x) \in$above));
$\forall x, y \in M\,(x{:}\,\sigma \wedge y : \tau \rightarrow (y, x) \in$above);
$\forall x, y \in M\,((x, x) \notin$ above);
$\forall x, y, z \in M\,((x, y) \in$above$\wedge (y, z) \in$above $\rightarrow (x, z) \in$above);
$\forall x, y \in M\,((x, y) \in$above $\rightarrow (y, x) \notin$above)
}

and the frame for **s** :

deframe : **s**
{
is-a : state
haspart : M = {a, b, c, d}
haspart : top = {a, c}
haspart : above = {(a, b), (a, D), (b, D), (c, d), (d, D)}
}

5 Conclusions and Further Work

Taking an ontology **O** as a structure, i.e., **O** = (O, ⊏), we define the corresponding logical theory T(**O**) as

$$T(\mathbf{O}) = \{\forall x\,(x{:}\,\alpha \rightarrow x{:}\,\beta), \exists y\,(y{:}\,\beta \wedge \neg y : \alpha) \mid \alpha \sqsubset \beta\} \cup \{T_\alpha \mid \alpha \in O\}.$$

We can prove by the quantifier elimination that any statement in **O** can be proved in T(**O**). Hence, any proposition holding in ontology **O** can be implied logically in T(**O**), and any proposition implied logically in T(**O**) is a proposition holding in **O**.

The transformations σ from an ontology to frames and τ from frames to logical theories, hence, the composition $\tau\sigma$ of σ and τ, are structural-property-preserving. This ensures that the representation of concepts in an ontology by using frames is faithful; and the transformation from frames to logical theories is faithful.

The definitions of ontologies and the subsumption relation (frames and the inheritance relation) compose of the presuppositions on ontologies (on frames), and under the presuppositions the faithfulness and fullness of the transformations are proved so that the inference in frame- or ontology-represented knowledge bases is correct.

References:

[1] Davis E. Representations of Commonsense Knowledge. Morgan Kaufmann Publisher, 1990.

[2] Nilsson NJ. Artificial Intelligence, A New Synthesis. Morgan Kaufmann Publisher, 1998.

[3] McCarthy J. First-Order theories of individual concepts and propositions. In: Hayes J, Michie D, Mikulich L, eds. Machine Intelligence 9. Ellis Horwood, Ltd., 1979. 129−147.

[4] Baader F, Calvanese D, McGuinness DL, Nardi D, Patel-Schneider PF. The Description Logic Handbook. Cambridge University Press, 2002.

[5] Sowa JF. Knowledge Representation: Logical, Philosophical, and Computational Foundations. Brooks Cole Publishing Co., 2000.

[6] Horty JF. Nonmonotonic logic. In: Goble L, ed. The Blackwell Guide to Philosophical Logic, Blackwell Philosophy Guides, Malden, Mass., Blackwell, 2001. 336−361.

[7] Cao CG, Feng QZ, Gao Y, Gu F, Si JX, Sui YF, Tian W, Wang HT, Wang LL, Zeng QT, Zhang CX, Zheng YF, Zhou XB. Progress in the development of national knowledge infrastructure. Journal of Computer Science and Technology, 2002,17(4):523−534.

[8] Gu F, Cao CG, Sui YF, Tian W. A domain-specific ontology of botany. Journal of Computer Science and Technology, 2004,19(2): 238−248.