Computer Aided Filament Winding for Elbows*

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Received March 1, 2001; accepted September 24, 2001

Abstract: Filament winding pattern plays a key role in computer-aided filament winding, unfortunately there is no reliable method for non-axisymmetric filament winding pattern design. In addition, a substantial amount of data storage is an obstacle to non-axisymmetric filament winding. A filament winding pattern for elbows and a new data processing method are presented in this paper.

Key words: composites; geometric pattern; algebraic pattern; node storage method; computer aided filament winding

Fiber composites represent a class of new materials providing high strength, lightweight and excellent chemical erosion resistance. Filament winding allows to place the fibers with a highly reproducible degree precision, and has therefore become a widely used technique for the production of high quality composite structure. Automation of the design and production can efficiently reduce labour-intensive design, development and cut down production costs. Meanwhile it can effectually control the quality of composite products. This requires the creation of a computer-integrated environment round the winding process, which includes design, production and quality control of the wound part, and therefore computer aided filament winding is a important technique in filament winding. The design and calculation of fiber paths and machine paths needs however considerable mathematical effort. The axisymmetric filament winding has been researched for several years and the production of the axisymmetric wound part such as revolving surface is relatively simple and mature. When winding an axisymmetric part, a basic fiber path is repeated continuously by indexing the winding axis, opposed to an asymmetric part, which consists of a number of different fiber paths that should be designed and computed individually resulting in a substantial amount of data. Furthermore, it is much harder to control all fiber paths to meet the technical requirements of filament winding such as stability, uniform coverage and non-bridging. Among the non-axisymmetric components, the elbow is a main product of composites. CADFIL[1] and CADWIND[2] are the only available software that is able to design and manufacture elbow and T-pieces part. Both adopted discrete method, which divided the mandrel surface into triangular pieces on which the fiber path corresponds to a straight line[1,2], and the computation of fiber path is relatively easy with only an additional processing for subdivision. Nevertheless, it is extremely difficult to estimate and control the error. It is actually a trial and error process[3].

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* Supported by the National Natural Science Foundation of China under Grant No.69823003 (国家自然科学基金)

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In China, a wound elbow was manually produced by mechanics-controlled filament winding machine. It is necessary to develop computer-aided filament winding (CAFW). CAFW is regarded as an inter-science and inter-discipline, which cover Mathematics, Computer science and Mechanics. Among these, mathematics is fundamental to fiber path design. In this paper, first we give a non-axisymmetric filament winding pattern (NFWP) for elbow based on geodesic, semi-geodesic and dwelling technique, and then a new data processing method called Node storage method is proposed which greatly reduces the amount of data storage. Finally, we develop an efficient algorithm for fiber path design on elbows.

1 Filament Winding Pattern

Filament winding is a process of weaving fiber around the mandrel, and the production of high performance composite structure needs an accurate placement of fiber path, hence the fiber path must be stable, that is to say the fiber should be fabricated in certain pattern. Mathematically, the pattern is called filament winding pattern (FWP), and it can be classified into two categories: geometric pattern and algebraic pattern[4]. The former is mainly involved in certain types of differential equations that specify the geometric characteristics of fiber paths, while the latter deals with the distribution of fiber paths around the mandrel. The design of fiber path is equivalent to the design of FWP that includes geometric pattern and algebraic pattern. Two kinds of patterns are not independent, and they must be consistent, and the algebraic pattern is greatly subject to the geometric pattern.

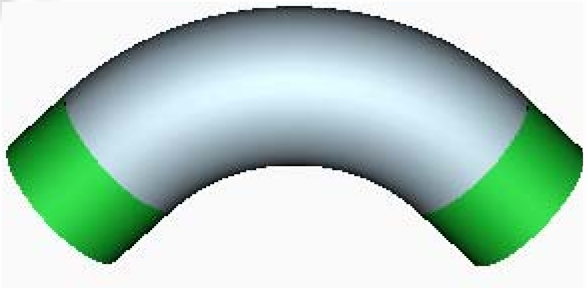

Fig.1 A mandrel with flange; gray for elbow and drak gray for flange

The elbow, as a joint part, has flanges on each end of it, and the flange can be considered as cylinder as shown in Fig.1. The differential equation of geodesic on elbow is easy to be solved, but it is hard to obtain the analytical form of semi-geodesic on the elbow. The system of differential equations of semi-geodesic can only be solved by numerical method, and the main problem is that we can't control it to meet requirement of full coverage, while it is convenient for us to get analytical solution of semi-geodesic on cylinder. A feasible NFWP is given by employing geodesic on elbow, semi-geodesic on cylinder and dwelling technique which trajectory dwells on the boundary circle instead of turning back immediately.

1.1 Geometric pattern

1.1.1 Geodesics on the elbow

To explain NFWP more clearly, we list some results of geodesic on elbow as follows.

$T(\theta,\varphi)$ is the vector representation of the elbow, and let $\theta_{\rm span}$ denote the span angle.

$$\boldsymbol{T}(\theta,\varphi)=\begin{pmatrix}(R+r\cos\varphi)\cos\theta\\ r\sin\varphi\\ (R+r\cos\varphi)\sin\theta\end{pmatrix},\ \theta\in\left[-\frac{\theta_{\rm span}}{2},\frac{\theta_{\rm span}}{2}\right],\ \varphi\in[0,2\pi]. \quad (1.1)$$

where θ and φ are the parameters of the elbow, as shown in Fig.2.

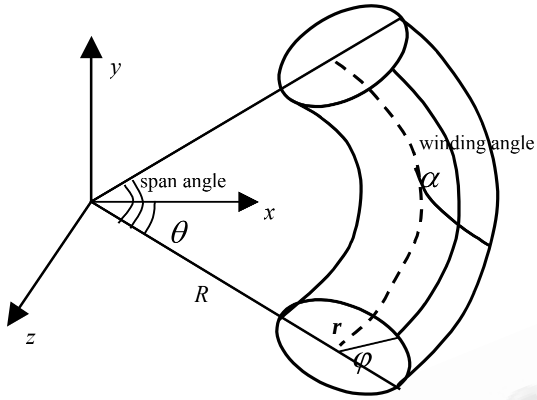

Fig.2 An elbow with parameters θ and φ

The first fundamental form of $T(\theta,\varphi)$ can be obtained by easy computation,

$$\sqrt{E}=R+r\cos\varphi,\ F=0,\ \sqrt{G}=r \tag{1.2}$$

and using Liouville formula, the geodesic curvature is expressed as

$$k_g=\frac{\mathrm{d}\alpha}{\mathrm{d}s}+\frac{\sin\varphi}{R+r\cos\varphi}\cos\alpha\ . \tag{1.3}$$

Let $k_g=0$, we obtain differential equations of geodesics as follows:

$$\frac{\mathrm{d}\alpha}{\mathrm{d}s}=-\frac{\sin\varphi}{R+r\cos\varphi}\cos\alpha\ , \tag{1.4}$$

$$\frac{\mathrm{d}\theta}{\mathrm{d}s}=\frac{\cos\alpha}{R+r\cos\varphi}\ , \tag{1.5}$$

$$\frac{\mathrm{d}\varphi}{\mathrm{d}s}=\frac{\sin\alpha}{r}\ . \tag{1.6}$$

Differential equations (1.4), (1.5) and (1.6) can be simplified as follows

$$\frac{\mathrm{d}\alpha}{\mathrm{d}\varphi}=-\frac{r\sin\varphi}{R+r\cos\varphi}\frac{\cos\alpha}{\sin\alpha}\ , \tag{1.7}$$

$$\frac{\mathrm{d}\theta}{\mathrm{d}\varphi}=-\frac{r}{R+r\cos\varphi}\frac{\cos\alpha}{\sin\alpha}\ . \tag{1.8}$$

From differential equation (1.7), we have

$$\sqrt{E}\cos\alpha=C\ . \tag{1.9}$$

where C is a constant value determined by initial wind angle α_0. This is well-known Clariaut equation.

The elbow is not always convex, even though the geodesics may bridge and may not cover the elbow unless the initial winding angle α_0 meet certain condition. Stable winding condition was given in Ref.[5] as follows:

$$0<\cos\alpha_0\le\frac{R-r}{R+r}\sqrt{\frac{R-r}{R}}\ . \tag{1.10}$$

1.1.2 Semi-Geodesics on the cylinder

In fact, the flange is a cylinder, so we only need design fiber path on cylinder and get the path on mandrel by simple coordinate transformation such as rotation and shift.

Let parametric surface of the cylinder C be

$$\boldsymbol{C}(z,\varphi)=\begin{pmatrix} r\cos\varphi \\ r\sin\varphi \\ z \end{pmatrix} \qquad 0\le\varphi<2\pi \tag{1.11}$$

where z and φ are two parameters of the cylinder as shown in Fig.3.

Similarly, we have E=1, F=0, G=r^2. Thereby, the parametric system (z,φ) is orthogonal, so we can use Liouville formula to compute geodesic curvature k_g

$$k_g=\frac{\mathrm{d}\alpha}{\mathrm{d}s}. \tag{1.12}$$

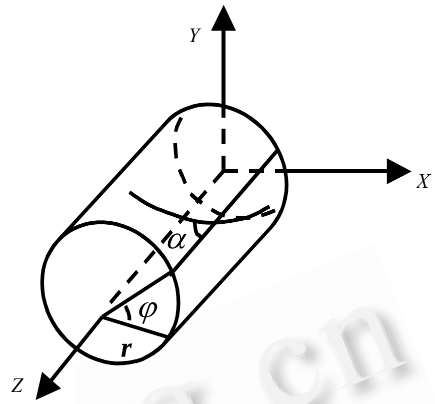

Fig.3 A cylinder with parameters z and φ

Normal curvature k_n is

$$k_n=-\frac{\sin^2\alpha}{r}. \tag{1.13}$$

λ is a slippage resistance, and meet $|\lambda|\le\mu$ the friction coefficient of fiber. Then semi-geodesics is defined by

$$\frac{k_g}{k_n}=\lambda. \tag{1.14}$$

From differential equations (1.12), (1.13) and (1.14), we derive the differential equations of semi-geodesic as follows:

$$\frac{\mathrm{d}a}{\mathrm{d}s}=-\frac{\lambda\sin^2 a}{r}. \tag{1.15}$$

In addition, we have

$$\frac{\mathrm{d}z}{\mathrm{d}s}=\cos\alpha, \tag{1.16}$$

$$\frac{\mathrm{d}\varphi}{\mathrm{d}s}=\frac{\sin\alpha}{r}. \tag{1.17}$$

Equation (1.15), (1.16) and (1.17) can be simplified as follows:

$$\frac{\mathrm{d}\alpha}{\mathrm{d}\varphi}=-\lambda\sin\alpha, \tag{1.18}$$

$$\frac{\mathrm{d}z}{\mathrm{d}\varphi}=r\cot\alpha. \tag{1.19}$$

(1.18) and (1.19) together with initial value

$$\alpha\Big|_{\substack{z=0\\ \varphi=\varphi_b}}=\alpha_b \tag{1.20}$$

and boundary condition

$$\alpha\Big|_{\substack{z=z_e\\ \varphi=\varphi_e}}=\frac{\pi}{2}, \tag{1.21}$$

where φ_b and α_b meet Clariaut equation (1.9), φ_e is unknown.

From (1.18), (1.19) and (1.20), we have

$$\tan\frac{\alpha}{2}=\tan\frac{\alpha_b}{2}e^{-\lambda(\varphi-\varphi_b)}. \tag{1.22}$$

$$z=\frac{r}{\lambda}\left(\frac{1}{\sin\alpha}-\frac{1}{\sin\alpha_b}\right).\tag{1.23}$$

1.2 Algebraic pattern

Algebraic pattern can be defined by axis motion in axisymmetric filament winding, but we can't simply apply the definition to non-axisymmetric filament winding. In theory, a precise definition of NFWP is given based on network[6]. The paper applies it to practical filament winding on elbow and proposes a practical NFWP.

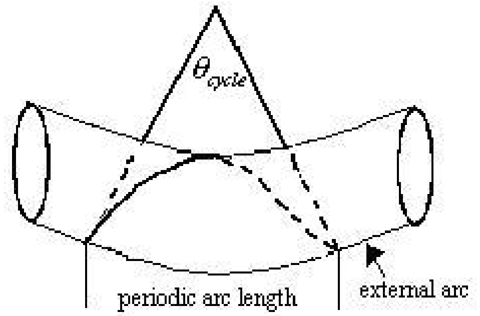

Fig.4 A geodesic within a period

Owing to the geometric feature of elbow, we cannot calculate the splitting number M on the transversal circle like common method. But we notice the following truth: the length of external arc defined by $\varphi=0$ on the elbow is the longest, so if fiber paths cover this longest arc, they must cover the whole mandrel. Considering periodicity of geodesics on elbow, we only calculate M within a period, illustrated in Fig.4.

Let w denote the tape width. From (1.8) and (1.9) we have

$$\theta_{cycle}=\int_0^{2\pi}\frac{r(R+r)\cos\alpha_0}{(R+r\cos\varphi)\sqrt{(R+r\cos\varphi)^2-(R+r)^2\cos^2\alpha_0}}\mathrm{d}\varphi,\tag{1.24}$$

where α_0 satisfies (1.10).

$$M=\left[\frac{(R+r)\theta_{\text{cycle}}\sin\alpha_0}{w}\right]+1,\tag{1.25}$$

where [ ] is the floor function.

Apparently, These tapes uniformly cover this periodic arc, when we place M tapes sequentially from $\theta=0$ to $\theta=\theta_{\text{cycle}}$ with interval $\frac{\theta_{\text{cycle}}}{M}$ along the external arc. We use the same number M determined by (1.25) on the cylinder.

Let $\{\theta_i\}$ $i=\overline{0,M-1}$ be a partition of θ_{cycle} defined by

$$\theta_i=\frac{i\theta_{\text{cycle}}}{M}\quad i=\overline{0,M-1}\tag{1.26}$$

and $\{\varphi_i\}$ $i=\overline{0,M-1}$ is a partition of 2π, and φ_i is defined by

$$\theta_i=\int_0^{\varphi_i}\frac{r(R+r)\cos\alpha_0}{(R+r\cos\varphi)\sqrt{(R+r\cos\varphi)^2-(R+r)^2\cos^2\alpha_0}}\mathrm{d}\varphi\ .\tag{1.27}$$

The distribution of fiber paths is completely determined by the starting point and the initial winding angle α_0 because of the uniqueness of geodesic, so we can only control the fiber distribution on cylinder to design ideal NFWP.

$T(0,0)$ is the starting point of the fiber path. Because the mandrel surface is symmetric about plane $z=0$, we only consider the positive half mandrel.

The number of φ_i when the fiber path passes through the half elbow from $\theta=0$ to $\theta=\frac{\theta_{\text{span}}}{2}$ is denoted by n_1 and n_2 denotes the number of φ_i when the fiber path passes through the half cylinder, and then the skipping number K is defined by

$$K \equiv 4n_1 + 4n_2 \pmod M \tag{1.28}$$

where n_1 is determined by α_0, so we should choose proper n_2 by designing semi-geodesic on cylinder in order to let M and K be relative prime.

From the Clariaut equation (1.9), we know that there are M different winding angles which are the initial winding angles of semi-geodesics when M different geodesics pass through the elbow to the flange. From (1.22) and (1.23), φ_e and λ are determined by α_b and z_e, so the value $\varphi_e-\varphi_b$ is different according to the different α_0. We use dwelling technique to deal with the hard problem such that $\varphi_e-\varphi_b \equiv n_2\frac{2\pi}{M}$.

2 Node Storage Method

Let $\{\theta_j\}$ be a partition of $\left[0,\frac{\theta_{\text{span}}}{2}\right]$ defined by

$$\theta_j=\frac{j\theta_{cycle}}{M}, j=\overline{0,l}, \tag{2.1}$$

where $l=\left[\frac{M\theta_{\text{span}}}{2\theta_{\text{cycle}}}\right]+1$.

Let $s_h(\varphi)$ $(h=\overline{0,M-1})$ be a semi-geodesic on cylinder, which is determined by (1.22), (1.23) and winding angle α_b^h, and the $\{\varphi_p^h\}$ $(p=\overline{0,n_2})$ be a partition of $\varphi_e^h-\varphi_b^h$ defined by

$$\varphi_p^h=\varphi_b^h+p\frac{\varphi_e^h-\varphi_b^h}{n_2} \tag{2.2}$$

where α_b^h and φ_b^h meet (1.9), and φ_e^h is determined by φ_b^h, α_b^h and z_e.

Along the semi-geodesic $s_h(\varphi)$, we define height partition of the cylinder $\{z_p^h=z_h(\varphi_p^h)\}$.

From (1.27), (2.1) and (2.2), the fiber path with starting point $T(0,0)$ consists of the point sequence $T(\theta_j,\varphi_i)$ on the elbow and $C(z_p^h,\varphi_p^h)$ on the cylinder, where $i\equiv j(\bmod M)$.

Definition 1. The intersection points between parametric curves $\varphi=\varphi_i$ and $\theta=\theta_j$ or between $\varphi=\varphi_p^h$ and $z=z_p^h$ is called node, where $i,h=\overline{0,M-1}$, $j=\overline{0,l}$, $p=\overline{0,n_2}$.

According to above result, we only calculate and save the values at nodes, so this method for data processing is called Node Storage Method. This method greatly lowers the computation cost and the amount of data storage, and it is very convenient for us to control quality of composites, we only need a simple processing for subdivision. The following table1 shows the efficiency of the method:

Table 1 Runtime of designing wound elbows

Winding parameters	Time of using CPU (s)
$\mu=0.2$ w=20mm α_0=78° θ_{span}= 90°	3
$\mu=0.2$ w=10 mm α_0=78° θ_{span}= 90°	5

3 Results

The NFWP designed in Section 2 and Node Storage Method have been successfully applied to the CAD system for filament winding, which is a main part of the important technology project of the national Ninth Five-Year-Plan of China. The following figures illustrate the filament winding on elbow. We'll perfect this software for composite material industry and produce sample of wound elbow this year. In the future research work, we will concentrate on filament winding on T-pieces.

Fig.5 A filament winding with $\mu = 0.2$, $\alpha_0 = 78°$, $w = 10$mm

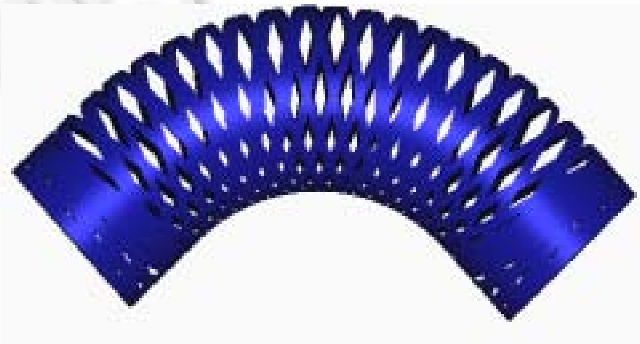

Fig.6 A winding pattern with μ=0.2, α_0=78°, w=15mm without mandrel

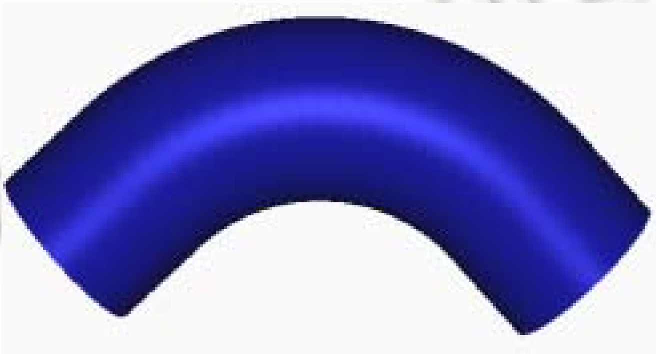

Fig.7 A finished wound elbow with $\mu = 0.2$, $\alpha_0 = 76°$, $w = 10$mm

Acknowledgement The authors wish to thank Professor Yu Yi-yue at the Department of Mathematics, Zhejiang University for his support to the project.

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用于弯管的计算机辅助纤维缠绕

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摘要: 纤维缠绕模式在计算机辅助纤维缠绕中起着关键的作用,然而对于非轴对称纤维缠绕设计,目前还没有稳定、可靠的模式设计方法.另外,如何有效地处理纤维路径设计中产生的大量数据也是非轴对称纤维缠绕的一大难题.给出了用于弯管的纤维缠绕模式和一种被称为节点存储法的新的数据处理方法.

关键词: 复合材料;几何模式;代数模式;节点存储法;计算机辅助纤维缠绕

中图法分类号: TP391 文献标识码: A