ISSN1000-9825 Journal of Software 软件学报 2000,11(7):881～888

Complexity Results for Restricted Credulous Default Reasoning*

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Abstract This paper concentrates on the complexity of the decision problem which decides whether a literal belongs to at least one extension of a default theory $\langle D, W\rangle$ in which D is a set of Horn defaults and W is a definite Horn formula or a Bi-Horn formula.

Key words Default logic, Horn default, credulous reasoning.

1 Introduction and Preliminaries

Default logic has been defined by Reiter in Ref. [1] and it is one of the most popular approaches to artificial intelligence for its ability to treat various forms of commonsense reasoning. However, a potential obstacle to use default logic is its high computational complexity. The three problems, that are most relevant in default logic and have been extensively studied in the literature, are deciding whether a default theory $\langle D, W\rangle$ has an extension, deciding whether a formula φ belongs to at least one extension of $\langle D, W\rangle$ (also known as credulous default reasoning) and deciding whether φ belongs to all the extensions (skeptical default reasoning). Generally, all the above decision problems are at the second level of the polynomial hierarchy. That is to say, propositional default reasoning is much harder than classical propositional reasoning. Loosely speaking, this additional complexity of default inference partly arises from its use of inference and consistency test. So, many attempts at suppressing the complexity of default reasoning are to study restrictions of the expressiveness of default theories where the inference and consistency checking is trivial or can be done in polynomial time (see Refs. [2～9]).

Kautz and Selman[7] prove that, as long as D is a set of Horn defaults (see the definition below) and W is a 1CNF formula, credulous reasoning for literals is solvable in linear time. However, the hypothesis that W is 1CNF cannot be fully relaxed, since Stillman[8] has proved that if W is a Horn formula then credulous reasoning is NP-complete. Then a natural question arises: when the expressiveness of W is enhanced between 1CNF and Horn, how does the complexity of credulous reasoning change? In this paper D always denotes a set of Horn defaults. We focus on the credulous reasoning of default theory $\langle D, W\rangle$ in which W is definite Horn, or Bi-Horn, or 2-Horn. The following results will be shown.

* This research is supported by the National Natural Science Foundation of China (国家自然科学基金, Nos. 19771045, 19931020) and the National High Technology Development Program of China (国家863高科技项目基金, No. 863-306-ZT06-01-2). **ZHAO Xi-shun** was born in 1964. He is a Ph.D. student at Department of Mathematics of Nanjing University. He became an associated professor of Henan Normal University in 1992. His research interests include artificial intelligence, theoretical computer science and mathematical logic. **DING De-cheng** was born in 1943. He is a professor and a doctoral supervisor of Department of Mathematics of Nanjing University and Yangzhou University. His research areas include recursive theory, artificial intelligence, theoretical computer science and multi-agent systems.

Manuscript received 1998-12-23, accepted 1999-05-10.

(1) When W is definite Horn, then credulous reasoning for positive literals can be solved in polynomial time. However, the reasoning for negative literals is still NP-complete.

(2) When W is Bi-Horn, then credulous reasoning for literals can be solved in polynomial time.

(3) When W is 2-Horn, then the reasoning is NP-complete.

Next we shall present some notations and preliminary results which will be used in the sequel sections.

Throughout this paper, the symbol $\mathscr{L}$ denotes the language of propositional logic. The symbols p,q,r,s,t (or indexed) are used for propositional atoms (also called positive literals). The symbols a,b,c,x,y,z (or indexed) are employed for literals (propositional atoms and their negations). The lower case Greek letters $\alpha,\beta,\gamma,\zeta,\eta$ (or with subscripts) are used for clauses (by a clause, we mean a disjunction of literals). And the symbols φ,ψ,θ (or indexed) are used for formulas. The sign $\sim$ is a meta-language operator that maps a positive literal to a negative one and vice versa.

A formula φ is in conjunctive normal form (CNF) if and only if it is a conjunction of clauses, i.e., $\varphi=\alpha_1\wedge\ldots\wedge\alpha_n$ with clauses $\alpha_i(1\leqslant i\leqslant n)$. We also consider a formula in CNF as a set of clauses. A clause is termed a positive (negative, respectively) clause if it contains no negative (positive, respectively) literals. A clause containing at most one positive (negative, respectively) literal is termed a Horn (dual Horn) clause. A clause is termed a k-clause if it contains at most k literals. A clause is referred to as a definite Horn clause if it contains exactly one positive literal. A formula φ in CNF is called a Horn (dual Horn, k-CNF, definite Horn) formula if it is a conjunction of Horn (dual Horn, k-, definite Horn, respectively) clauses. A formula is called 2-Horn if it is both Horn and 2CNF. A formula φ is called Bi-Horn if it is Horn and dual Horn. Obviously, every 1CNF formula is a Bi-Horn formula and every Bi-Horn formula is a 2CNF formula. The following results are well known.

Proposition 1. Every definite Horn formula is satisfiable.

Proposition 2. The satisfiability of Horn formulas is decidable in linear time.

Proposition 3. Given a Horn formula φ and a literal x, $\varphi\vdash x$ can be determined in polynomial time.

Proposition 4. The satisfiability problem for 3CNF formulas is NP-complete.

A default is a rule of the form

$$\frac{\varphi:\psi}{\theta}$$

where φ,ψ,θ are formulas. φ is called the prerequisite of the default, ψ is called justification, and θ is the consequence. Given a default δ, we write $p(\delta)$ for the prerequisite of δ, $j(\delta)$ for its justification, and $c(\delta)$ for its consequence. Given a set D of defaults, define

$$p(D)=\{p(\delta)\mid\delta\in D\},$$
$$j(D)=\{j(\delta)\mid\delta\in D\},$$
$$c(D)=\{c(\delta)\mid\delta\in D\}.$$

A default is normal if its justification and consequence are the same. A (normal) default theory is a pair $\langle D,W\rangle$ where D is a set of (normal) defaults and W is a set of clauses (intuitively, W is called initial knowledge). In a Horn default, the prerequisite is a conjunction of positive literals, the justification and consequence are the same literal, That is, the default has the form

$$\frac{p_1\wedge\ldots\wedge p_n:y}{y}.$$

Given a default theory $\langle D,W\rangle$ in which D is a set of Horn defaults, then $\langle D,W\rangle$ is termed a definite Horn (Bi-Horn, 2-Horn, respectively) default theory if W is definite Horn (Bi-Horn, 2-Horn, respectively).

Definition 1. Given a default theory $\langle D,W\rangle$ and a theory S, S is an extension of $\langle D,W\rangle$ if and only if it satisfies the following equations:

$$E_0=W,$$

$$E_{i+1}=Cn(E_i)\cup\left\{\theta\,\middle|\,\frac{\varphi:\psi}{\theta}\in D,\ \varphi\in E_i \text{ and } \neg\psi\notin S\right\},$$

$$S=\bigcup_{i=1}^{\infty}E_i,$$

where $Cn(E_i)$ is the deductive closure of E_i.

Finally, we recall default proof theory which will be used later.

Definition 2. Let $\langle D,W\rangle$ be a normal default theory, and let $\delta_1,\ldots,\delta_n$ be a sequence of defaults from D. We say that $\delta_1,\ldots,\delta_n$ is a default proof of a formula φ if

(1) $W\cup c(\{\delta_1,\ldots,\delta_n\})$ is consistent,

(2) $W\cup c(\{\delta_1,\ldots,\delta_n\})\vdash\varphi$,

(3) for every $1\leqslant i\leqslant n$, $W\cup c(\{\delta_j: j<i\})\vdash p(\delta_i)$.

Theorem 1. Let $\langle D,W\rangle$ be a normal default theory and φ a formula. Then φ appears in some extension of $\langle D,W\rangle$ if and only if φ has a default proof.

Usually only minimal default proofs are of interest, in other words, default proofs from which we cannot delete any further defaults without losing the property. Then we have a default proof of the required formula.

2 Credulous Reasoning of Definite Horn Default Theories

Proposition 5. Given a consistent set H of Horn clauses and a negative clause α such that $H\cup\{\alpha\}$ is also consistent, then for any positive literal p,

$$H\vdash p \text{ if and only if } H\cup\{\alpha\}\vdash p.$$

Proof. ($\Rightarrow$): Trivial.

($\Leftarrow$): Recall that the class of Horn formulas is closed under deduction, i.e., if φ is a Horn formula and γ is a non-tautological clause such that $\varphi\vdash\gamma$ then there is a subclause ρ of γ such that $\varphi\vdash\rho$ and ρ is Horn. Now suppose that $H\cup\{\alpha\}\vdash p$. Since $H\cup\{\alpha\}$ is consistent, there is a negative literal x occurring in α such that $H\cup\{x\}$ is consistent. Clearly, $H\cup\{x\}\vdash p$. Hence $H\vdash\sim x\vee p$. Since $\sim x\vee p$ is not Horn, it must be that $H\vdash\sim x$ or $H\vdash p$. Noticing that $H\vdash\sim x$ is impossible, we have $H\vdash p$.

Proposition 6. Given a consistent set H of dual Horn clauses and a positive clause α such that $H\cup\{\alpha\}$ is also consistent, then for any positive literal p,

$$H\vdash\neg p \text{ if and only if } H\cup\{\alpha\}\vdash\neg p.$$

Proof. Similar to the proof of Proposition 5.

Lemma 1. Let $\langle D,W\rangle$ be a definite Horn default theory, p a positive literal. Define $H=W\cup H_1$, where H_1 is the following set of Horn clauses:

$$H_1=\left\{\alpha\supset y\,\middle|\,\frac{\alpha:y}{y}\in D,\ W\nvdash\sim y,\text{ and } y \text{ is positive}\right\}.$$

Then p appears in some extension of $\langle D,W\rangle$ if and only if $H\vdash p$.

Proof. ($\Rightarrow$): Suppose that p appears in some extension S of $\langle D,W\rangle$. Let

$$GD(S)=\{\delta\in D\mid S\vdash p(\delta)\text{ and } c(\delta)\in S\},$$

and let

$$HGD(S)=\left\{\alpha\supset y\,\middle|\,\frac{\alpha:y}{y}\in GD(S)\right\}.$$

Then S is the unique extension of $\langle GD(S),W\rangle$. It is not difficult to see that $S=Cn(W\cup HGD(S))$. Thus $W\cup HGD(S)\vdash p$. By Proposition 5, we have $H_p\vdash p$, where

$$H_p=\{\gamma\in W\cup HGD(S)\mid\gamma\text{ is a definite Horn clause}\}.$$

Clearly, $H_s \subset H$. Hence $H \vdash p$.

($\Leftarrow$): Suppose $H \vdash p$. Since H is a set of definite Horn clauses, by Proposition 1, we know that H is consistent. Now let

$$\gamma_1, \gamma_2, \ldots, \gamma_n$$

be one proof of p. Let

$$\gamma_{i_1}, \gamma_{i_2}, \ldots, \gamma_{i_k}$$

be the clauses generated by defaults from D. Clearly, $W \cup \{\gamma_{i_1}, \ldots, \gamma_{i_k}\}$ is consistent and

$$W \cup \{\gamma_{i_1}, \ldots, \gamma_{i_k}\} \vdash p.$$

Replace each γ_{i_j} by the corresponding default rule. It is clear that the resulting sequence is a default proof of p. Theorem 1 implies that p appears in some extension of $\langle D, W \rangle$.

Remark. The idea of the proof of Lemma 1 is in fact the same as that of Lemma 6.4 in Ref. [7]. However, Lemma 1 generalizes Lemma 6.4 in Ref. [7] to the case in which the initial knowledge W is definite Horn. Lemma 1 shows that this generalization only applies to positive literals.

Notice that the problem of determining if a literal follows from a Horn formula can be solved in polynomial time. It is not difficult to see that H can be defined in polynomial time. Therefore, we have the theorem below.

Theorem 2. The problem of deciding if a given positive literal appears in some extension of a definite Horn default theory is polynomial.

Then the following question naturally arises: is there a deterministic polynomial algorithm to solve credulous reasoning of negative literals when $\langle D, W \rangle$ is definite Horn? Unless P is NP, the answer is generally no. In fact 3SAT can be reduced to credulous reasoning of a single negative literal (see Theorem 3 below).

Theorem 3. Determining if a given negative literal belongs to some extension of a definite Horn default theory is NP-complete.

Proof. Now we define a reduction from 3CNF to defaults. Let φ be a propositional 3CNF formula. Then φ can be written in the following form:

$$\alpha_1 \wedge \ldots \wedge \alpha_k \wedge \beta_1 \wedge \ldots \wedge \beta_m \wedge \eta_1 \wedge \ldots \wedge \eta_n,$$

such that each α_i, $1 \leqslant i \leqslant k$, is a Horn clause; each β_i, $1 \leqslant i \leqslant m$, is of the form $s_i \vee t_i \vee \neg e_i$; and each η_i, $1 \leqslant i \leqslant n$, is of the form $p_i \vee q_i \vee r_i$, where $s_i, t_i, e_i, p_i, q_i, r_i$ are positive literals.

For each β_i, pick two new atoms u_i and t_{β_i}. For each η_i, pick a new atom t_{η_i}. Finally, pick a new atom p.

Let $W = \{\alpha_1, \ldots \alpha_k\}$, $W' = W \cup \{\neg u_i \vee \neg e_i \mid 1 \leqslant i \leqslant m\}$. And let $W'' = \{\gamma \in W' \mid \gamma \text{ is definite Horn}\} \cup \{p \vee \gamma \mid \gamma \in W', \gamma \text{ contains no positive literal}\}$. Clearly, W'' is a set of definite Horn clauses.

Now let D be made up of the following groups of defaults:

(A) For each β_i, $1 \leqslant i \leqslant n$, the rules:

$$\frac{:s_i}{s_i}, \frac{:\neg s_i}{\neg s_i}, \frac{:t_i}{t_i}, \frac{:\neg t_i}{\neg t_i}, \frac{:u_i}{u_i}, \frac{:\neg u_i}{\neg u_i}, \frac{:e_i}{e_i}, \frac{:\neg e_i}{\neg e_i}, \frac{s_i : t_{\beta_i}}{t_{\beta_i}}, \frac{t_i : t_{\beta_i}}{t_{\beta_i}}, \frac{u_i : t_{\beta_i}}{t_{\beta_i}}.$$

(B) For each η_i, $1 \leqslant i \leqslant n$, the rules:

$$\frac{:p_i}{p_i}, \frac{:\neg p_i}{\neg p_i}, \frac{:q_i}{q_i}, \frac{:\neg q_i}{\neg q_i}, \frac{:r_i}{r_i}, \frac{:\neg r_i}{\neg r_i}, \frac{p_i : t_{\eta_i}}{t_{\eta_i}}, \frac{q_i : t_{\eta_i}}{t_{\eta_i}}, \frac{r_i : t_{\eta_i}}{t_{\eta_i}}.$$

(C) The single rule:

$$\frac{t_{\beta_1} \wedge \ldots \wedge t_{\beta_m} \wedge t_{\eta_1} \wedge \ldots \wedge t_{\eta_n} : \neg p}{\neg p}.$$

Our theorem directly follows from the following claim:

Claim. φ is satisfiable if and only if W is consistent and $\neg p$ appears in some extension of $\langle D, W''\rangle$.

Proof of the claim.

($\Rightarrow$): Suppose that φ is satisfiable. Since each u_i, $1 \leqslant i \leqslant m$, is a new atom, it is easy to see that

$$\{\alpha_1, \ldots, \alpha_k, \beta_1 \ldots, \beta_m, \eta_1, \ldots, \eta_n\} \cup \{u_i \equiv \neg e_i \ 1 \leqslant i \leqslant m\}$$

is also satisfiable. This implies that

$$W'' \cup \{\beta_1 \ldots, \beta_m, \eta_1, \ldots, \eta_n\} \cup \{u_i \vee e_i \mid 1 \leqslant i \leqslant m\} \cup \{\neg p\}$$

is consistent since p is a new atom. Notice that $t_{\beta_1}, \ldots, t_{\beta_m}, t_{\eta_1}, \ldots, t_{\eta_n}$ are also new atoms. It follows that the formula

$$W'' \cup \{\beta_1, \ldots, \beta_m, \eta_1, \ldots, \eta_n\} \cup \{u_i \vee e_i \mid 1 \leqslant i \leqslant m\} \cup \{t_{\beta_1}, \ldots, t_{\beta_m}, t_{\eta_1}, \ldots, t_{\eta_n}\} \cup \{\neg p\},$$

denoted as S, is consistent, too. Then let v be a valuation such that $v(S)=1$. Put

$$E = \{x \mid x \text{ or } \sim x \text{ occurs in } S \text{ and } v(x)=1\}.$$

It is not difficult to verify that $Cn(E)$ is an extension of $\langle D, W''\rangle$.

($\Leftarrow$): Suppose that W is consistent and suppose that E is an extension such that $\neg p \in E$. Since W is consistent, W' is consistent. Furthermore, $\neg p$ is not provable from W''. Thus $\neg p$ must be obtained by applying the default in group (C). Consequently, $t_{\beta_1}, \ldots, t_{\beta_m}, t_{\eta_1}, \ldots, t_{\eta_n} \in E$. This implies that for every β_i, $s_i \in E$ or $t_i \in E$ or $u_i \in E$ and that for every η_i, $p_i \in E$ or $q_i \in E$ or $r_i \in E$. It follows that

$$\{\beta_1, \ldots, \beta_m, \eta_1, \ldots, \eta_n\} \in E.$$

Consequently, $\varphi \in E$. Since W'' is definite Horn, it is consistent. Hence E is consistent. This implies that φ is satisfiable.

3 Credulous Reasoning of Bi-Horn Default Theories

In this section, we study the complexity of Bi-Horn default theories.

Theorem 4. The problem of determining whether a positive literal appears in some extension of a Bi Horn default theory is solvable in polynomial time.

Proof. Let $\langle D, W\rangle$ be a Bi-Horn default theory, p a positive literal. Define H to be the set $W \cup H_1$, where H_1 is the following set of Horn clauses:

$$H_1 = \left\{\alpha \supset y \mid \frac{\alpha : y}{y} \in D, W \nvdash \sim y, \text{ and } y \text{ is positive}\right\}.$$

Our theorem follows from the claim:

Claim. p appears in some extension of $\langle D, W\rangle$ if and only if $H \vdash p$.

Proof of claim.

($\Rightarrow$): Suppose that p appears in some extension of $\langle D, W\rangle$. If $W \vdash p$ then the result follows. So suppose that $W \nvdash p$. By Theorem 1, pick a minimal default proof $\Delta = \delta_1, \ldots, \delta_n$ of p. Since every δ_i is Horn, by Proposition 5 and the minimality of Δ, we know that each $c(\delta_i)$ is positive. It follows that $p(\delta_i) \supset c(\delta_i) \in H_1$. Therefore, $H \vdash p$.

($\Leftarrow$): Suppose that $H \vdash p$. If $W \vdash p$ then p appears in every extension of $\langle D, W\rangle$. Thus suppose that $W \nvdash p$. Notice that W is Bi-Horn. Then W is dual Horn. By Proposition 6, it follows that $W \cup \{y \mid$ there is α such that $(\alpha \supset y) \in H_1\}$ is consistent. Thus $W \cup H_1$ is consistent. The rest proof is the same as the second part of Lemma 1.

Now we consider credulous reasoning of a negative literal when $\langle D, W\rangle$ is a Bi-Horn default theory.

Definition 3. Let $\langle D, W\rangle$ be a finite Bi-Horn default theory, x a negative literal. Define

$$H_x=\{\alpha\supset y\in H_1 \mid W\cup\{y\}\nvdash\sim x\},$$

$$p(x,D,W)=\{p \mid p \text{ is positive and } W\cup H_x\vdash p\},$$

where H_1 is the set defined in the proof of Theorem 4.

Proposition 7. Let X be a set of 2CNF clauses. x and y are two literals such that $X\cup\{x,y\}$ is consistent. Then for any literal z, $X\cup\{x,y\}\vdash z$ if and only if $X\cup\{x\}\vdash z$ or $X\cup\{y\}\vdash z$.

Proof. (⇐): Trivial.

(⇒): Suppose that $X\cup\{x,y\}\vdash z$. Then $X\vdash\sim x\vee\sim y\vee z$. Since the class of 2CNF clauses is closed under deduction, it follows that

$$X\vdash\sim x\vee\sim y \text{ or } X\vdash\sim x\vee z \text{ or } X\vdash\sim y\vee z.$$

Because $X\cup\{x,y\}$ is consistent, $X\vdash\sim x\vee\sim y$ is impossible. Consequently, $X\cup\{x\}\vdash z$ or $X\cup\{y\}\vdash z$.

Lemma 2. Let $\langle D,W\rangle$ be a Bi-Horn default theory, x a negative literal. If W is consistent then $P(x,D,W)\nvdash\sim x$.

Proof. Suppose that $P(x,D,W)\vdash\sim x$. Then $\sim x$ is provable from $W\cup\{y \mid \alpha\supset y\in H_x$ for some $\alpha\}$ which is consistent by Proposition 6. By Proposition 7, there must be some y such that $W\cup\{y\}\vdash\sim x$ and that $\alpha\supset y\in H_x$ for some α. This contradicts the definition of H_x.

Definition 4. Given a Bi-Horn finite default theory $\langle D,W\rangle$, define

$$N(D,W)=\left\{x \,\middle|\, \begin{array}{l} x \text{ is a negative literal, } W\nvdash\sim x \\ \text{and there is a } \delta\in D \text{ such that} \\ p(\delta)\subseteq P(x,D,W) \text{ and } c(\delta)=x \end{array}\right\}.$$

Lemma 3. Let $\langle D,W\rangle$ be a Bi-Horn default theory. Then every x in $N(D,W)$ appears in some extension of $\langle D,W\rangle$.

Proof. By the definition of $N(D,W)$, pick $\delta\in D$ such that $x=c(\delta)$ and $p(\delta)\subseteq P(x,D,W)$. Then by the proof of Theorem 4, every y in $p(\delta)$ has a default proof Δ_y such that the consequences of all defaults in Δ_y are positive. Since W is Bi-Horn, by Proposition 6, combining all these Δ_y's we get a default proof Δ. It is easy to see that Δ,δ is a default proof of x. Then by Theorem 1, there is an extension E such that $x\in E$. This completes our proof.

Lemma 4. Let $\langle D,W\rangle$ be a Bi-Horn default theory, y a negative literal. Then y appears in some extension of $\langle D,W\rangle$ if and only if there is $x\in N(D,W)$ such that $W\cup\{x\}\vdash y$.

Proof. (⇐): Directly from Lemma 3.

(⇒): Suppose that y appears in some extension of $\langle D,W\rangle$. If $W\vdash y$ then the result follows. So suppose that $W\nvdash y$. By Theorem 1, pick a minimal default proof $\Delta=\delta_1,\ldots,\delta_n$ of y. By Propositions 6 and 7 and the minimality of Δ, there is exactly one i such that $W\cup\{c(\delta_i)\}\vdash y$. Again by the minimality of Δ, it follows that $c(\delta_n)$ is negative and that $W\{c(\delta_n)\}\vdash y$. It is not difficult to see that $c(\delta_n)\in N(D,W)$.

Theorem 5. There is polynomial algorithm which determines if a negative literal appears in some extension of a Bi-Horn default theory.

Proof. To decide if a literal appears in some extension of $\langle D,W\rangle$ we can use the algorithm shown in Fig. 1. This algorithm is based on Lemma 4. In this algorithm there are at most n iterations of the first **for** loop. In each such iteration, to compute H_x and $P(x,D,W)$ it needs $O(n^2)$ time. To check whether $p(\delta)\subseteq P(x,D,W)$ it also needs $O(n^2)$ time. Therefore, it takes $O(n^3)$ time to compute $N(D,W)$. It is easy to see that the algorithm needs $O(n^2)$ time to check if there is some $x\in N(D,W)$ such that $W\cup\{x\}\vdash y$. The total time for the algorithm is therefore $O(n^3)$.

From Theorems 4 and 5 we see that credulous reasoning with a literal and a Bi-Horn default theory is polynomial. However the result will become false when we replace "Bi-Horn" by "2-Horn".

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Input: A finite Bi-Horn default theory (D,W) and a negative lietral y
Output: Answer to the question if y appears in some extesion of (D,W)
N(D,W) := ∅
for δ ∈ D such that c(δ) is negative do
      x := c(δ)
      compute H_x
      compute P(x,D,W)
      check that W ⊬ ~x and that p(δ) ⊆ P(x,D,W)
      if this condition holds then N(D,W) := N(D,W) ∪ {x}
rof
for x ∈ N(D,W) do
      check that W ∪ {x} ⊢ y
      if this condition holds then return YES
rof
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Fig. 1 Credulous reasoning algorithm with a negative literal and a Bi-Horn default theory

Theorem 6. Determining if a given literal belongs to some extension of a 2-Horn default theory is NP-complete.

Proof. Now we define a reduction from 3CNF to defaults. Let the function π map each positive literal to itself, and each negative literal $\neg p$ to a new literal p'. Consider any propositional formula φ in 3CNF. Then φ can be written in the form: $\alpha_1 \wedge \ldots \wedge \alpha_n$ such that each α_i is of the form $x_i \vee y_i \vee z_i$, $1 \leqslant i \leqslant n$. For each i, pick a new atom t_i. Finally, pick new atom t. Let

$$W = \{\neg \pi(x) \vee x \mid x \text{ is a negative literal occurring in } \varphi\}.$$

Clearly, W is a set of 2-Horn clauses. Now let D be made up of the following groups of defaults:

(A) For each i, $1 \leqslant i \leqslant n$, the rules:

$$\frac{:x_i}{x_i}, \frac{:\neg x_i}{\neg x_i}, \frac{:y_i}{y_i}, \frac{:\neg y_i}{\neg y_i}, \frac{:z_i}{z_i}, \frac{:\neg z_i}{\neg z_i}, \frac{:\pi(x_i)}{\pi(x_i)}, \frac{:\neg \pi(x_i)}{\neg \pi(x_i)}, \frac{:\pi(y_i)}{\pi(y_i)}, \frac{:\neg \pi(y_i)}{\neg \pi(y_i)}, \frac{:\pi(z_i)}{\pi(z_i)}, \frac{:\neg \pi(z_i)}{\neg \pi(z_i)}.$$

(B) For each i, $1 \leqslant i \leqslant n$, the rules:

$$\frac{\pi(x_i):t_i}{t_i}, \frac{\pi(y_i):t_i}{t_i}, \frac{\pi(z_i):t_i}{t_i}.$$

(C) The single rule:

$$\frac{t_1 \wedge \ldots \wedge t_n : t}{t}.$$

Our theorem directly follows from the following claim:

Claim. φ is satisfiable if and only if t appears in some extension of $\langle D,W \rangle$.

Proof of Claim. Leave to the readers.

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若干限制形式的缺省推理的复杂性

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摘要 该文研究判定一文字是否出现在缺省理论(D,W)的某一扩张中的复杂性.其中,D是一集Horn缺省规则,而W是definite Horn公式或者Bi-Horn公式.

关键词 缺省逻辑,Horn缺省规则,轻信推理.
中图法分类号 TP18