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On Liveness and Safeness for Weighted Extended Free Choice Nets*

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Abstract Liveness and safeness are important behavioral properties of nets (systems). Many powerful results have been derived for some subclasses of Place/Transition nets (systems). The aim of this paper is to draw a general perspective of the liveness and safeness for a subclass with Extended Free Choice net-like underlying graph but allowing weights: Weighted Extended Free Choice nets (WEFC nets). First, a brief and intuitive proof of liveness equivalent condition for WEFC nets is given. Then, for safe nets, a sufficient and necessary condition is presented.

Key words Weighted extended free choice net, live, safe, structurally live, structurally safe.

Place/Transition net (P/T net)[1] is a mathematical tool well suited for modelling and analyzing systems exhibiting behaviors such as concurrency, conflict and causal dependency between events. However, the high degree of complexity of the analysis limits the applicability of Petri nets to real-world problems. The reachability graph of such systems is actually unmanageable, thus it is crucial to enforce the analysis power of techniques based on the net structure. This paper presents new results in this direction.

Liveness and safeness are main behavioral properties of P/T nets. Liveness corresponds to the absence of global or local deadlock situations, safeness to the absence of overflows. Many results of liveness and safeness theory have been derived for restricted classes of P/T nets[2~6], due to the use of some graph theoretic tools. We wondered how much it would change if we allow weights in the model. We consider the so-call WEFC nets (weighted extended free choice nets), that generalize the well known class of Extended Free Choice nets. In this paper, we first give an interesting simple proof to the liveness theorem (A WEFC net is live if and only if every subsystem generated by siphon is live). Then, for Safe WEFC nets, we give an equivalent condition.

The paper is organized as follows. The next section presents the basic concepts and notations. In section 2, the liveness for the WEFC nets is investigated. In section 3, we present a sufficient and necessary condition of (structure) liveness and (structure) safeness in WEFC nets. Section 4 concludes the paper.

1 Basic Concepts

We assume the reader is familiar with the structure, firing rule and basic properties of net models[7], and

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with elementary graph theory. However, in this section we recall some basic concepts and notations to be used.

Definition 1.1.

1. A (Petri) net is a triple $N=(P,T;F)$ where
   (1) $P=\{p_1,p_2,\ldots,p_m\}$ is a finite set of places,
   (2) $T=\{t_1,t_2,\ldots,t_n\}$ is a finite set of transitions,
   (3) $F\subseteq(P\times T)\cup(T\times P)$ is a set of arcs (flow relation),
   (4) $P\cap T=\varnothing$ and $P\cup T\neq\varnothing$,
   (5) $dom(F)\cup cod(F)=P\cup T(dom(F)=\{x\mid\exists\ y:(x,y)\in F\}$, $cod(F)=\{x\mid\exists\ y:(y,x)\in F\})$.
   $N=(P,T;F,W)$ is a weighted (Petri) net, where $(P,T;F)$ is a (Petri) net, $W:F\to\{1,2,3,\ldots\}$ is a weight function.
2. A pair of a place p and a transition t is called a self-loop iff $(p,t)\in F\wedge(t,p)\in F$. A net is said to be pure iff it has no self-loops.

In the following, we'll only consider pure nets.

Definition 1.2. Let $N=(P,T;F,W)$ be a weighted net.

1. The incidence matrix A of N is an $m\times n$ matrix of integers and its entry is given by

$$a_{i,j}=\begin{cases}W(t_j,p_i) & \text{if } p_i\in t_j^{\bullet}\\ -W(p_i,t_j) & \text{if } p_i\in {}^{\bullet}t_j\\ 0 & \text{otherwise}\end{cases}$$

2. A net C is an elementary circuit iff it is connected and $\forall\ u\in P\cup T:|{}^{\bullet}u|=|u^{\bullet}|=1$. Since $|P|=|T|=m>1$ here, we adopt the following notations: ${}^{\bullet}p_i=\{t_i\}$ and $p_i^{\bullet}=\{t_{i\oplus1}\}$, $W(t_i,p_i)=c_{i,i}\in Z^+$ (Z^+ is the set of nonzero positive integers), $W(p_i,t_{i\oplus1})=c_{i,i\oplus1}\in Z^+$, where $1\leqslant i\leqslant m$, $i\oplus1\overset{\text{def}}{=}$ if $i=m$ then 1 else $i+1$. The gain of an elementary circuit C is $\prod_C\overset{\text{def}}{=}\prod_{j=1}^{m}(c_{j,j}/c_{j,j\oplus1})$. C is neutral, absorbing or generating when the value of $\prod_C$ equals, is less than or is greater than 1 respectively.
3. $N=(P,T;F)$ is WSM (weighted state machine) iff $\forall\ t\in T:|{}^{\bullet}t|=|t^{\bullet}|=1$.
4. $N=(P,T;F)$ is weighted T-system iff $\forall\ p\in P:|{}^{\bullet}p|=|p^{\bullet}|=1$.
5. N is WEFC net iff
   (1) $\forall\ p_1,p_2\in P:p_1^{\bullet}\cap p_2^{\bullet}\neq\varnothing\Rightarrow p_1^{\bullet}=p_2^{\bullet}$,
   (2) $\forall\ p\in P,\forall\ t_1,t_2\in p^{\bullet}:W(p,t_1)=W(p,t_2)$.
6. A net N is strongly connected iff $\forall\ x,y\in P\cup T$, there exists a directed path from x to y. A net N is P-connected iff $\forall\ x,y\in P$, there exists a directed path from x to y.
7. $\forall\ t_1,t_2\in T$, t_1 is called in structure conflict with t_2 iff ${}^{\bullet}t_1\cap{}^{\bullet}t_2\neq\varnothing$.

Definition 1.3.

1. A net system is a 2 tuple $\Sigma=(N,M_0)$ where
   (1) N is a weighted net; (2) $M_0:P\to\{0,1,2,3,\ldots\}$ is the initial marking.
2. The set of all markings reachable from M_0 is called the reachability set and denoted by $R(N,M_0)$.
3. $t\in T$ is live iff $\forall\ M\in R(N,M_0)$, $\exists\ M'\in R(N,M)$: t is enabled at M'.
   $t\in T$ is dead iff t can never be fired at any $M\in R(N,M_0)$.
4. A place p in the preset of t is called enablingly marked iff $M(p)\geqslant W(p,t)$.
5. (N,M_0) is safe iff $\exists\ k\in Z^+$, $\forall\ p\in P$, $M\in R(N,M_0)$: $M(p)\leqslant k$. (Safe is the recommended term in Net Community, sometimes it is called bounded).
   N is structurally safe iff $\forall\ M_0$, (N,M_0) is safe.
6. (N,M_0) is live iff $\forall\ t\in T$, t is live.

N is structurally live iff $\exists\ M_0$, (N,M_0) is live.

$N=(P,T;F)$ is a deadlock at marking M_c iff $\forall\ t\in T$, t is dead.

7. Let $\Sigma=(N,M_0)$ be a WEFC system. For $\forall\ p\in P$, define $\bar{p}$ as follows: If $p^{\cdot}\neq\varnothing$, then for $t\in p^{\cdot}$, $\bar{p}=W(p,t)$ else $\bar{p}=\infty$.

   Divide P into two subsets under any reachable marking $M\in R(M_0)$:

   $P^M_{<}=\{p\mid p\in P\wedge M(p)<\bar{p}\}$

   $P^M_{\geq}=\{p\mid p\in P\wedge M(p)\geq\bar{p}\}$

Property 1.1.

1. $P^M_{<}\cap P^M_{\geq}=\varnothing$, $P^M_{<}\cup P^M_{\geq}=P$.
2. If a siphon $H\subseteq P^M_{<}$, then N' generated by H is a deadlock under M, i.e., $\forall\ M'\in R(N,M)$, $\forall\ t\in H^{\cdot}$, t is not enabled at M'.
3. A WEFC system $\Sigma(N,M)$ is a deadlock $\Leftrightarrow\forall\ p\in P$: $M(p)<\bar{p}$.

In the following, M^H, $M|_H$ and $\sigma|_H$ denote a marking of siphon H under system Σ_H that is generated by H, a marking of siphon H under system (N,M) and a firing sequence only including elements in $H^{\cdot}$ respectively.

2 Liveness

In this section, we discussed the equivalent relation between a WEFC system and its subsystem generated by its siphon. References[8,9] gave the same result for WFC nets (weighted free choice nets). Their proofs are different but both are very long and complicated. Here we give not only a proof for a larger class of WFC systems, WEFC systems, but also a quite simple proof for WEFC system in order to show some useful proof techniques. First, we introduce a lemma.

Lemma 2.1. Let $\Sigma=(P,T;F,W,M_0)$ be a WEFC system and t be a transition of N. t is not live iff there exist a siphon H containing a place $p\in{}^{\cdot}t$ and a reachable marking M such that $H\subseteq P^M_{<}$.

Proof. If such a siphon exists, t is obviously not live.

Now consider the necessary condition by recurrence on the number of transitions of the net.

If Σ is a system with only one transition, the lemma holds (If t is not live, one of its input places $p\in P^M_{<}$ and this place is a siphon).

Let Σ be a system with at least two transitions, t be a transition that is not live. As t is not live, there exists a reachable marking M such that t is dead under M. There are only two cases:

Case 1. In Σ, there exists another transition, u, that is not live.

By applying the recurrence hypothesis for transition u on the system $(N\backslash\{t\},M)$, we obtain a marking M' reachable from M and a siphon H_u, such that $H_u\subseteq P^{M'}_{<}$, $\dot{u}\cap H_u\neq\varnothing$. Transitions t and u are dead for the system (N,M'), so for any reachable marking of this system, we have $H_u\subseteq P^{M'}_{<}$.

By applying the recurrence hypothesis for transition t on the system $(N\backslash\{u\},M')$, we obtain a marking M'' reachable from M' and a siphon H_t, such that $H_t\subseteq P^{M''}_{<}$, that contains a place in the preset of t.

Now $H_t\cup H_u$ is a siphon in (N,M_0) which contains a place in the preset of t, and we have a marking $M''\in[M_0>$, such that $H_t\cup H_u\subseteq P^{M''}_{<}$.

Case 2. There exists no other transition in (N,M) that is not live.

Case 2.1. t is not in structure conflict with another transition.

Hence, each time a place p in the preset of t is enablingly marked, it remains enablingly marked until t fires. As t is dead under M, there must exist a place p in the preset of t such that p is never enablingly marked from M onward. If the preset of p contains no transition, the $\{p\}$ is the siphon needed. Else, all the transitions in the preset of p are dead under $M' \in R(M)$.

Then it contradicts the assumption that only t is not live.

Case 2.2. t is in structure conflict with another transition v.

From the definition of WEFC net, v is not live under M, this contradicts the assumption.

That is the proof. □

Theorem 2.1. If every subsystem generated by siphon in a WEFC system Σ is live, then the WEFC system Σ is live.

Proof. Assume Σ is not live, then there at least exists a transition t which is not live. From Lemma 2.1, there exist a siphon H containing a place $p \in {}^{\bullet}t$ and a reachable marking M such that $H \subseteq P^{M}_{<}$. As firing of t ($\forall t \in T \backslash H^{\bullet}$) can't change the marking of H, the firing sequence $\sigma|_H$ can be fired in Σ_H (Σ_H is the system generated by H). Now we have $M_0^H[\sigma|_H > M_1^H$ and $H \subseteq P^{M_1^H}_{<}$. This contradicts the liveness of Σ_H. □

Theorem 2.2. If a WEFC system is live, then every subsystem generated by sipnon is live.

Proof. Assume $N=(P,T;F,W)$ is a WEFC net, system $\Sigma(N,M_0)$ is live, but there exists a siphon $H' \subseteq P$ such that subsystem $\Sigma'=(N',M'_0)$ generated by H' is not live, i.e., there exists a transition t which is not live in Σ'. From Lemma 2.1, there exist a siphon H containing a place $p \in {}^{\bullet}t$ and a reachabele marking M'_1, such that $H \subseteq H'^{M'_1}_{<}$, $M'_0[\sigma' > M'_1$. As firing of t ($\forall t \in H'^{\bullet}$ but $t \notin H^{\bullet}$) can't change the marking of H and the firing sequence $\sigma'|_H$ can be fired in Σ_H (Σ_H is the system generated by H).

Let $\sigma'|_H = t_1 \ldots t_n$ ($n \geqslant 0$). If we can prove that there exists a σ, such that $M_0[\sigma > M_1$ and $H \subseteq P^{M_1}_{<}$, there will be a contradiction with the liveness of Σ. The theorem holds.

In the following, we will operate by recurrence on the number of n to prove the existence of σ and M_1.

Case 1. If $n=0$, then let $\sigma=\varnothing$, $M_1=M_0$, the theorem holds.

Case 2. If $n>0$, we have two subcases:

Case 2.1. Let $u \in H^{\bullet}$ be a transition enabled under M_0 in $\Sigma|_H$, hence u is enabled under M'_0 in $\Sigma|_H$. Let $p \in H$ and $u \in p^{\bullet}=\{v_1,\ldots,v_f\}$ (if $f=1$, then $v_1=u$), then there at least exists a transition $t_i \in \{t_1,\ldots,t_n\}$ in $\{v_1,\ldots,v_f\}$. Let t_i be the first element of $p^{\bullet}$, which first appears in $\sigma'|_H$. From the definition of WEFC net, we have $t_i \cap (\bigcup_{j=1}^{i-1} {}^{\bullet}t_j)=\varnothing$ and can rewrite $\sigma'|_H = t_i t_1 \ldots t_{i-1} t_{i+1} \ldots t_n$. Thus, we have $M'_0[t_i > M'_2[t_1 \ldots t_{i-1} t_{i+1} \ldots t_n > M'_1$, $M_0[t_i > M_2$ and $M_2|_{\Sigma_H}=M'_2$.

As $n-1<n$, applying the recurrence hypothesis on M_2, M'_2, $t_1 \ldots t_{i-1} t_{i+1} \ldots t_n$, there exist σ_1 and M_3 such that $M_2[\sigma_1 > M_3$ and $H \subseteq P^{M_3}_{<}$. Let $\sigma = t_i \sigma_1$, $M_1=M_3$, the theorem holds.

Case 2.2. No transition in $H^{\bullet}$ is enabled under M_0 in Σ.

As Σ is live, let u be the first enabled transition in $H^{\bullet}$ under M_2 ($M_0[\sigma_1 > M_2$). Obviously, $\sigma_1 \cap H^{\bullet}=\varnothing$. Hence we have $M_2|_{\Sigma_H}=M'_0$.

Let $p \in H$ and $u \in p^{\bullet}=\{v_1,\ldots,v_f\}$, then there at least exists a transition $t_i \in \{t_1,\ldots,t_n\}$ in $\{v_1,\ldots,v_f\}$. Let t_i be the first element of $p^{\bullet}$, which first appears in $\sigma'|_H$. From the definition of WEFC net, we have $t_i \cap (\bigcup_{j=1}^{i-1} {}^{\bullet}t_j)=\varnothing$ and can rewrite $\sigma|_H = t_i t_1 \ldots t_{i-1} t_{i+1} \ldots t_n$. Thus, we have $M'_0[t_i > M'_2[t_1 \ldots t_{i-1} t_{i+1} \ldots t_n > M'_1$, $M_2[t_i > M_3$ and $M_3|_{\Sigma_H}=M'_2$.

As $n-1<n$, applying the recurrence hypothesis on M_3, M'_2, $t_1 \ldots t_{i-1} t_{i+1} \ldots t_n$, there exist σ_2 and M_4 such that $M_3[\sigma_2 > M_4$ and $H \subseteq P^{M_4}_{<}$. Let $\sigma=\sigma_1 t_i \sigma_2$, $M_1=M_4$, the theorem holds. □

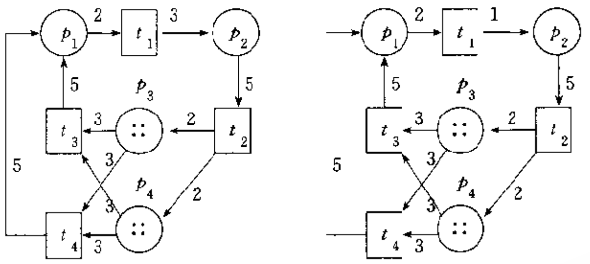
From Theorem 2.1 and Theorem 2.2, we have the following theorem.

Theorem 2.3. A WEFC system is live iff every subsystem generated by sipnon is live.

Corollary 2.1. A WEFC system is live iff every subsystem generated by minimal sipnon is live.

Proof. The necessary condition is obvious.

Assume a WEFC system $\Sigma=(P,T;F,W,M_0)$ is not live. From Lemma 2.1, there must exist a siphon H, σ and $M_0[\sigma>M$ such that $H\subseteq P^M_<$. As in H, there at least exists a minimal siphon H', then $H'\subseteq P^M_<$. This contradicts the liveness of the subsystem generated by minimal siphon H'. □



(a) A live WEFC system　(b) A not live WEFC system

Fig. 1

Figure 1(a) shows that if every subsystem generated by the siphons $\{p_1,p_2,p_3\}$, $\{p_1,p_2,p_4\}$ is live, then the WEFC system is live. In Figure 1(b), as the subsystem generated by siphon $\{p_1,p_2,p_3\}$ or $\{p_1,p_2,p_4\}$ is not live, so the WEFC system is not live.

From now on, the liveness analysis of a WEFC system is much easier than before as it is enough to concentrate on the minimal siphon liveness only, which is a very small system analysis.

3 Liveness and Safeness

Safeness in Petri nets is a very important and useful behavioral property and based on Petri nets, some international traffic safety standards are going to be made public. That is the motivation of this section.

Lemma 3.1. If a WEFC system $\Sigma=(P,T,F,W,M_0)$ is live and safe, then $\forall\ p\in P$, p must be included in a (minimal) siphon.

Proof. Let p be a place of P, such that p is not included in any (minimal) siphon. As Σ has no isolated place, p at least has one input transition $t\in T$ (if p has no input transition, Σ is not live) and $p\in t^{\cdot}$. Let $M_1\in R(M_0)$ and $M_1(p)$ be the maximal marking of p (Σ is safe).

Consider marking M'_1, such that $M'_1(p)=0$ and $M'_1(q)=M_1(q)$ $(q\neq p)$. As every subsystem generated by (minimal) siphon is live under M_1 (from Theorem 2.2), every subsystem generated by (minimal) siphon is live under M'_1. So, from Theorem 2.1, the WEFC system Σ is live under M'. Therefore, we can reach a marking M'_2 such that $M'_2(p)\neq 0$. We can define $M_2(p)=M'_2(p)+M_1(p)$ and $M_2(q)=M'_2(q)$ $(q\neq p)$. So $M_2\in R(M_1)$, moreover, $M_2\in R(M_0)$. But $M_2(p)>M_1(p)$, this is in contradiction with the safeness of p. The lemma holds. □

In Fig. 2, as p_5 is not included in any siphon, firing t_4 will increase the number of tokens consistently. So the WEFC system is unsafe.

Theorem 3.1. A WEFC system $\Sigma=(P,T,F,W,M_0)$ is live and safe iff

(i) every subsystem generated by siphon is live and safe;

(ii) $\forall\ p\in P$, p must be included in a siphon.

Proof. ⇦ Assume the WEFC system is unsafe, then, there at least exists an unsafe place p. From (ii), we know p must be included in a siphon H (let M^H_0 be the initial marking of H, Σ_H be the subsystem generated by siphon H).

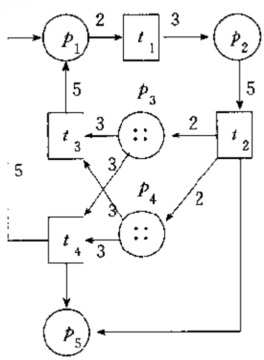


Fig. 2　An unsafe but live WEFC system

As Σ_H is safe, let $K(p)$ be the upper bound of p. The WEFC system is unsafe, so there must exist a firing sequence σ and M_1 $(M_0[\sigma>M_1)$, such that $M_1(p)>K(p)$.

As firing of t $(\forall\ t\in T\backslash H^{\cdot})$ can't change the marking of H, the firing sequence $\sigma|_H$ can be fired in Σ_H.

Now we have $M_0^H[\sigma|_H>M_1^H$ and $M_1|_H=M_1^H$, so $M_1^H(p)>K(p)$. This contradicts the safeness of Σ_H. So the WEFC system is safe.

From Theorem 2.1, the WEFC system is live. Sufficient conditions hold.

⇒ From Theorem 2.2, every subsystem generated by siphon is live.

From Lemma 3.1, $\forall p\in P$, p must be included in a siphon.

Let $K(p)$ be the upper bound of p in Σ. Assume there exists an unsafe subsystem Σ_H generated by siphon H, then from the liveness of Σ, there exists a firing sequence $\sigma_H=t_1\ldots t_n$ such that $\forall t\in H^{\cdot}: t\in\{t_1,\ldots,t_n\}$, $M_0^H[\sigma_H>M_1^H$ and $M_1^H(p)>K(p)$. If we can prove that there exist a firing sequence σ and marking M_1, such that $M_0[\sigma>M_1$ and $M_1(p)>K(p)$, there will be a contradiction with the safeness of Σ. Safeness of every subsystem generated by siphon holds.

In the following, we will operate by recurrence on n to prove the existence of σ and M_1.

Case 1. If $n=0$, then let $\sigma=\varnothing$, $M_1=M_0$, the safeness of subsystem holds.

Case 2. If $n>0$, we have two subcases as follows:

Case 2.1. Let u ($u\in H^{\cdot}$) be a transition enabled under M_0 in Σ, hence, u is enabled under M_0^H in Σ_H. Let $p\in H$ and $u\in p^{\cdot}=\{v_1,\ldots,v_f\}$ (if $f=1$, then $v_1=u$). Let t_i be the first element of $p^{\cdot}$, which first appears in σ_H. From the definition of WEFC net, we have $t_i\cap(\bigcup_{j=1}^{i-1}{}^{\cdot}t_j)=\varnothing$ and can rewrite $\sigma_H=t_it_1\ldots t_{i-1}t_{i+1}\ldots t_n$. Thus, we have $M_0^H[t_i>M_2^H]t_1\ldots t_{i-1}t_{i+1}\ldots t_n>M_1^H$, $M_0[t_i>M_2$ and $M_2|_H=M_2^H$.

As $n-1<n$, applying the recurrence hypothesis on M_2, M_2^H, $t_1\ldots t_{i-1}t_{i+1}\ldots t_n$, there exist σ_1 and M_3 such that $M_2[\sigma_1>M_3$ and $M_3(p)>K(p)$. Let $\sigma=t_i\sigma_1$, $M_1=M_3$, the safeness of subsystem holds.

Case 2.2. No transition in $H^{\cdot}$ is enabled under M_0 in Σ.

As Σ is live, we can let u be the first enabled transition in $H^{\cdot}$ under M_2($M_0[\sigma_1>M_2$). Obviously, $\sigma_1\cap H^{\cdot}=\varnothing$. Hence we have $M_2|_H=M_0^H$. This transforms to Case 2.1. Safeness of subsystem holds. □

Theorem 3.2. A WEFC system $\Sigma=(P,T,F,W,M_0)$ is live and safe iff

(i) every subsystem generated by minimal siphon is live and safe;

(ii) $\forall p\in P$, p must be included in a minimal siphon.

Proof. ⇒ Follow Theorem 3.1 and Lemma 3.1.

⇐ Assume the WEFC system is unsafe, then, there at least exists an unsafe place p. From (ii), p must be included in a minimal siphon H (let $M_0{}^H$ be the initial marking of H, Σ_H be the subsystem generated by minimal siphon H). As Σ_H is safe, we let $K(p)$ be the upper bound of p. The WEFC system is unsafe, so there must exist a firing sequence σ and M_1($M_0[\sigma>M_1$.

As firing of t ($\forall t\in T\setminus H^{\cdot}$) can't change the marking of H, the firing sequence $\sigma|_H$ can be fired in Σ_H. Having $M_0^H[\sigma|_H>M_1^H$ and $M_1|_H=M_1^H$ must have $M_1^H(p)>K(p)$. This contradicts the safeness of Σ_H. So the WEFC system is safe.

From Corollary 2.1, we know the WEFC system is live. Sufficient conditions hold. □

From Theorem 3.1 and Theorem 3.2, we have the following corollary.

Corollary 3.1. A WEFC net $N=(P,T;F,W)$ is structurally live and structurally safe iff

(i) every subsystem generated by (minimal) siphon is structurally live and structurally safe;

(ii) $\forall p\in P$, p must be included in a (minimal) siphon.

From Corollary 3.1, the structure liveness and structure safeness of WEFC net transform to the structure liveness and structure safeness of minimal siphon, which is very small net structure analysis.

Lemma 3.2. H is a minimal siphon in WEFC net iff (a) H is P-connected, (b) $\forall t\in H^{\cdot}$, $|{}^{\cdot}t\cap H|=1$.

Note: In [7], this lemma is for the WFC net. By the same method, we can have this lemma.

Lemma 3.3. Let H be a minimal siphon in the WEFC net. If H is live under M, then $\forall M_i\geq M$ H is live under M_i.

Proof. From Ref. [8], we have the above conclusion for the WFC net. As H is a WFC net, Lemma 3.3 holds. □

Lemma 3.4. [8] Every elementary circuit C is structurally live iff C is neutral or generating.

Lemma 3.5. [9] If a connected Petri net system (N, M_0) is live and safe, then N is strongly connected.

Lemma 3.6. [10] Let (N, M_0) be a live Weighted T-system. If $M'_0 \geq M_0$ then (N, M'_0) is live.

Lemma 3.7. [7] A place p in a Petri net N is structurally unsafe iff there exists an n-vector X of nonnegative integers such that $AX = \Delta M \gneq 0$, where the pth entry of $\Delta M > 0$ (i.e., $\Delta M(p) > 0$).

Proposition 3.1. Let $N = (P, T; F, W)$ be a weighted net. If N is strongly connected, then any $x \in P \cup T$ must be included in some elementary circuits.

Theorem 3.3. Let H be a minimal siphon in the WEFC net. If H is structurally live and structurally safe, then every elementary circuit must be neutral or absorbing.

Proof. Assume there exists an elementary circuit $C(t_0 p_0 \ldots t_n p_n t_0)$ such that $\prod_C = \frac{c_{0,0} \cdots c_{n,n}}{c_{0,1} \cdots c_{n,0}} > 1$ and let A_C be the incident matrix of C. Now we only concentrate on the analysis of C. Let an n-vector $X^T = \left[1, \frac{c_{0,0}}{c_{0,1}}, \ldots, \prod_{j=0}^{n-1} \frac{c_{j,j}}{c_{j,j\oplus 1}}\right]$, then we have $A_C \left(\left(\prod_{j=0}^{n-1} c_{j,j\oplus 1}\right) X\right) = \left[0, \ldots, 0, c_{n,0}\left(\left(\prod_{j=0}^{n-1} c_{j,j\oplus 1}\right)\left(\prod_{j=0}^{n} \frac{c_{j,j}}{c_{j,j\oplus 1}}\right) - 1\right)\right]^T$. As $\prod_{j=0}^{n} \frac{c_{j,j}}{c_{j,j\oplus 1}} > 1$, therefore $A_C\left(\left(\prod_{j=0}^{n-1} c_{j,j\oplus 1}\right) X\right) \gneq 0$. Because $A_C\left(\left(\prod_{j=0}^{n} c_{j,j\oplus 1}\right) X\right) \gneq 0$ and $\left(\prod_{j=0}^{n} c_{j,j\oplus 1}\right) X$ is a vector of nonnegative integers, from Lemma 3.7, we know C is structuraly unsafe. Therefore, there exist a marking M_C of C, a firing sequence σ_C and a $\bar{p} \in C$, such that $M_C[\sigma_C > M'_C$ and $M'_C(\bar{p}) > K(\bar{p})$ ($K(\bar{p})$ is any bound of p). As H is live, there exists a living marking M_H. Define marking M'_H, such that $\forall\, p \in H$ $M'_H(p) = M_H(p)$ (if $p \in C$ and $M_C(p) < M_H(p)$ or p is not included in C), $M'_H(p) = M_C(p)$ (if $p \in C$ and $M_C(p) \geq M_H(p)$). From Lemma 3.3, H is live under M'_H. As H is structurally safe, $\bar{p}$ must have an upper bound $\tilde{K}(\bar{p})$. As H is a minimal siphon in WEFC net, from Lemma 3.2, we have $\forall\, t \in H^\bullet$, $|{}^\bullet t \cap H| = 1$. Therefore, the firing sequence σ_C can be fired in subsystem Σ_H and we have $M'_H[\sigma_C > M''_H$ such that $\bar{p} > \tilde{K}(\bar{p})$. This contradicts the safeness of $\bar{p}$. Theorem 3.3 holds. □

Theorem 3.4. Let H be a minimal siphon in the WEFC net, Σ_H be the subsystem generated by H and every elementary circuit is neutral. If H is a structurally live and structurally safe, then H is a structurally live, structurally safe and strongly connected WSM.

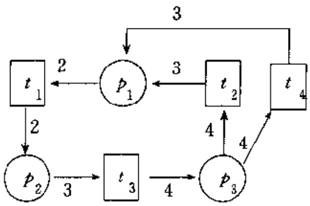
Proof. As H is structurally safe and structurally live, from Lemma 3.5, Σ_H is strongly connected.

As H is a minimal siphon, from Lemma 3.2, we have $\forall\, t \in H^\bullet$, $|{}^\bullet t \cap H| = 1$.

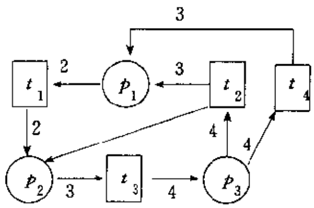
Assume there exists a transition $t \in H^\bullet$ such that $t^\bullet > 1$. Let $t^\bullet = \{p_1, \ldots, p_m\}$ $(p_1, \ldots, p_m \in H)$. From Proposition 3.1, we know t must be included in an elementary circuit C and let $p_1 \in C$. As $\prod_C = 1$, from Lemma 3.4, C is structurally live. Therefore, there exists a living marking M_C of C. As H is live, there exists a living marking M_H. Define marking M'_H, such that $\forall\, p \in H$ $M'_H(p) = M_H(p)$ (if $p \in C$ and $M_C(p) < M_H(p)$ or p is not included in C), $M'_H(p) = M_C(p)$ (if $p \in C$ and $M_C(p) \geq M_H(p)$). From Lemma 3.3, H is live under M'_H. From Lemma 3.6, C is live under M'_H. We can only fire transitions in C and firing t will consistently increase the marking of $p_2, \ldots, p_m$. This contradicts the safeness of H. So $\forall\, t \in H^\bullet$, $|t^\bullet \cap H| = 1$. Thus, H is a structurally live, structurally safe and strongly connected WSM.

In Figure 3(a), the minimal siphon is a structurally live and safe WSM, so the minimal siphon is structurally live and safe. But in Figure 3(b), $t_2{}^\bullet > 1$, therefore the minimal siphon is structurally unsafe.

So for a minimal siphon in which every elementary circuit is neutral, the problem of structure liveness and



(a) Structurally live and safe



(b) Structurally live but unsafe

Fig. 3 Two minimal siphons in WEFC net

safeness is transformed to structure liveness and safeness of WSM in which every elementary circuit is neutral. Here we'll not discuss neutral WSM in detail. In case of existing absorbing circuits in the minimal siphon, the problem will be very complicated, i. e., further discussion is needed.

4 Conclusion

We present a quite simple proof of liveness equivalent condition for WEFC nets. Then, for safe WEFC nets, a sufficient and necessary condition is presented. We conjecture the living problem for safe WEFC nets can be decided in polynomial time, which is a real challenging problem to attract further study.

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论加权扩充自由选择网的活性与安全性

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摘要 活性与安全性是网系统重要的行为性质.对于位置/变迁网系统的一些子类,人们已为之导出许多有意义的结果.该文贡献在于为一种称为加权扩充自由选择网的网系统子类找到活性与安全性的一般性质.文章先给出其活性等价条件的简要与直观的证明,随后,对于这一类安全的子系统又给出活性的充分必要条件.

关键词 加权扩充自由选择网,活,安全,结构活,结构安全.

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