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Wavelength Conversion in All-Optical Bi-Directed Networks*

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Abstract: In many models of optical routing, a set of communication paths (requests) in a network are given, and a wavelength must be assigned to each path so that paths sharing an edge receive different wavelengths. The goal is to assign as few wavelengths as possible, in order to make as efficient use as possible of the optical bandwidth. Much work in the area has considered the use of wavelength converters: if a node of a network contains a converter, any path passing through this node may change its wavelength. Having converters at some of the nodes can reduce the number of wavelengths down to congestion bound. Thus Wilfong and Winkler defined a set S of nodes to be sufficient if, placing converters at the nodes in S, every set of paths can be routed with a number of wavelengths equal to its congestion bound. In this paper, the minimum sufficient set problem in bi-directed networks is studied. The problem is transformed into minimum vertex cover problem and some algorithms are developed for the problem.

Key words: approximation algorithm; WDM network; wavelength conversion; vertex cover; sufficient set

The assignment of wavelengths to communication paths (requests) is a basic optimization problem for optical networks based on wavelength division multiplexing (WDM)[1,2]. The basic model for an optical WDM network consists of a graph with nodes and edges. Communication requests from source nodes to destination nodes are to be carried out by paths in the graph connecting each pair of source and destination. Two such paths may share some edges in the graph. In this case, the message set between these two paths must be transmitted with different wavelengths. Given a set of such paths, the problem here is to decide whether this set of requests can be carried out using a limited number of wavelengths.

The problem can be modeled in the following standard way. We are given a directed graph G, and a set P of paths in G, and wish to assign a color (or wavelength) to each path in P, so that no two paths sharing an edge receive the same color; we will call such a wavelength assignment *valid*. The goal is to minimize the number of colors used in a valid assignment; we will denote this minimum by $\chi(P)$.

Typically, WDM networks have been thought of in two broad categories. In a wavelength selective (WS) network the edges in the route assigned to a communication path must all allocate the same wavelength to that path whereas in a wavelength interchanging (WI) network the edges in the route assigned to a communication path may allocate different wavelengths to the path. Clearly, the nodes in a WI network require some sort of hardware that

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takes incoming signals on the different wavelengths and permutes them for the outgoing signals. We call such a device a *wavelength converter*. Placing a wavelength converter at a node v enables any path containing v to change its color as it passes through v. In a network with converters, our notion of a valid wavelength assignment must become more general: it is now an assignment of a wavelength to each *edge* of each path, with the restriction that the sequence of color assignments to a path can only change when the path passes through a converter. Note that if we were to place a converter at every node of G. the minimum number of colors required in a valid assignment would be equal to the following natural *congestion bound* $\upsilon(P)$: the maximum number of paths passing through any single edge. In the absence of converters, $\chi(P)$ can be arbitrarily larger than $\upsilon(P)$ on some instances; even with converters, we cannot get away with fewer than $\upsilon(P)$ colors because a different color is required for each path that passes through the most congested edge.

Since wavelength converters are expensive components of a network, one would like to use them as parsimoniously as possible while still achieving a substantial reduction in wavelength usage. Motivated by this, Wilfong and Winkler[3] defined a subset S of the nodes of a graph to be *sufficient* if it has the following property: with converters at the nodes in S, any set P paths has a valid wavelength assignment with only $\upsilon(P)$ colors. They then proposed the following basic network design problem: Given a graph G, find a sufficient set of minimum size. They proved this problem to be NP-complete, even on planar graph[3].

Obviously, in the absence of converters, any set P can be allocated a valid wavelength assignment with $\upsilon(P)$ colors if a network is a bi-directed path. Wilfong and Winkler[3] considered the problem for a ring network. They showed that there is a polynomial time algorithm that will realize a valid assignment with at most $2\upsilon(P)$ wavelengths without wavelength conversion. LI Guo-jun *et al*. improved the performance ratio by designing an approximation algorithm that assigns the wavelengths no more than $3/2(\upsilon(P))$. If the ring network has wavelength converters, Wilfong and Winkler showed that only one node of the ring needs to place a converter.

In this paper we mainly study sufficient set in bi-directed tree networks (Section 2) and general bi-directed networks (Section 3). Fortunately, this problem can be transformed into vertex cover problem. We only need to study the algorithm for the minimum vertex cover problem.

1 Preliminaries

A network, in this paper, is a bi-directed graph $G=(V,E)$. For each edge $e=\{u,v\}\in E$ there correspond two directed links (u,v) and (v,u) of the network. The *skeleton* of the network G, denoted $s(G)$, is the undirected graph obtained from G by replacing each bi-directed pair of links by a single undirected edge. For the sake of simplicity, we will assume that $s(G)$ is connected for all networks G that we deal with. We will also assume that G does not contain multiple copies of the same edge in the same direction.

A *routing* R is a collection of directed paths. Let P denote a set of directed paths in G. We will assume throughout that no individual path in P passes through any vertex more than once. The congestion of P, denoted $\upsilon(P)$, is defined to be the maximum, over all the directed edges e of G, of the number of paths containing e. The *conflict graph* of P is an undirected graph L whose vertices are the set of paths in P; two paths P_i and P_j are joined by an edge in L if and only if they share a link in G. The minimum number of wavelengths(colors) in a valid wavelength assignment for P denoted $\chi(P)$, is simply the chromatic number of the conflict graph of P .

2 Sufficient Sets in Bi-Directed Trees

In this section, we begin with a description of the efficient algorithm for bi-directed trees.

We now look at a type of subgraph that necessitates a converter. Consider a graph consisting of a bi-directional path with at least two bi-directed pairs of links at each of its two ends. For concreteness, let the path be P and its ends be u and v. Suppose that we have two bi-directed pairs of links at each of its ends, i.e., u_1 and u_2 are adjacent to u ($u_1 \neq u_2$). Similarly, v_1 and v_2 are adjacent to v ($v_1 \neq v_2$). Let us call such a graph an *H -graph*. The bi-directional path P of an H-graph will be called its *characteristic path*.

As shown in Fig.1, one can place 5 paths in an H-graph whose conflict graph is a 5-cycle. So $\upsilon(P)=2$ and $\chi(P)=3$. Thus we have

Lemma 1. Let G be a bi-directed tree, S a sufficient set for G, and K an H-graph in G with characteristic path P. Then $S \cap P \neq \varnothing$.

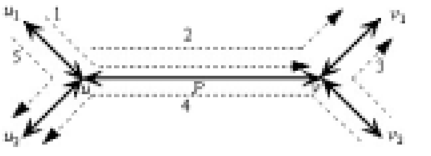

Fig.1 An H-graph

Wilfong and Winkler[3] have defined a *spider* to be a tree with at most one vertex of degree greater than 2 and given the following theorem.

Theorem 1[3]. The empty set is sufficient for G if and only if G is a spider.

Now let us show that H-graphs are the structures that necessitate converters when $s(G)$ is a tree.

Lemma 2. Suppose $s(G)$ is a tree, then the empty set is sufficient for G if and only if G has no H-graph.

Proof. By Lemma 1, we know that if the empty set is sufficient, then G has no H-graph. It is easy to prove the converse by Theorem 1. □

If K and K' are H-graphs in a bi-directed tree G, we say that K' *encloses* K if the characteristic path of K is a subset of the characteristic path of K'. Enclosure, defined in this way, imposes a partial order on the H-graphs of G, and we say an H-graph is *minimal* if it is minimal with respect to enclosure. We say that a node of a path P is an *internal node* if it is not one of the two endpoints. The following lemma is obvious.

Lemma 3. (i) If K is a minimal H-graph in G, with characteristic path P, then there is no edge of $G \backslash K$ incident on an internal node of P. (ii) The characteristic paths of two minimal H-graphs are edge-disjoint.

We now use some definitions from Ref.[3]. For a subset $S \subseteq V$, define the graph $G_s(S)$ as follows. The nodes $V(S)$ of $G_s(S)$ are the nodes in $V \backslash S$ together with pairs $\langle s,e \rangle$ for each $s \in S$ and each edge e incident to s in $s(G)$. The edges of $G_s(S)$ consist of the edges $\{u,v\}$ of $s(G)$ where $u, v \notin S$, together with $\{\langle s,e \rangle, v\}$ whenever $e=\{s,v\}$ and $\{\langle s,e \rangle, \langle t,e \rangle\}$ whenever s and t are adjacent nodes of S. We may think of $G_s(S)$ as the result of *splitting* each node s of S into degree-of-s-many copies.

We are now ready to prove the following characterization theorem.

Theorem 2. Let G be a bi-directed tree, then a set of nodes S in G is sufficient if and only if it intersects characteristic path of each minimal H-graph.

Proof. Lemma 2 implies that S must meet the characteristic path of each minimal H-graph. Conversely, suppose that S has this property, and consider the graph G' obtained by *splitting* G (as above) at each node in S. Each component X of G' has the property that it contains no H-graph, and $s(X)$ is also a tree. By Lemma 2, we get the result. □

By Lemma 3, the characteristic paths of the minimal H-graphs in G form a collection of edge-disjoint bi-directional paths $\{P_i\}$, each of whose internal nodes has degree 2 in $s(G)$. Given any sufficient set S for G, we can therefore transform it into a sufficient set S' of no greater size that meets each P_i at one of its ends.

Let G' denote the graph obtained from G by replacing each of the P_i by a single bi-directed edge e'_i between its endpoints, and deleting the internal nodes and the other bi-directed edges that are not in the characteristic paths of the minimal *H*-graphs. As a consequence of Theorem 2, our problem is equivalent to finding a vertex set of minimum size that intersects each of the edges in $s(G')$. Obviously e'_i is the edge set of G' and $s(G')$ is a forest. So this problem can be directly reduced to the standard vertex cover problem in a forest. We now present an efficient Greedy algorithm. The algorithm relies on the observation that every tree has at least two vertices of degree 1, and a vertex of degree 1 never needs to be included in an optimal cover, because the adjacent vertex may always be included instead without loss of optimality.

Algorithm 1.

Input: $s(G')$ and $S \leftarrow \varnothing$;

While vertices of degree 1 remain {

  select a vertex of degree 1;

  add its adjacent vertex to S;

  remove this vertex , its adjacent vertex and all the edges incident to them from $s(G')$;

}

Output: S.

3 Sufficient Sets in General Bi-Directed Graphs

In general bi-directed graphs, Kleinberg and Kumar[4] have given a description of the 2-approximation algorithm. Here we want to give a further discussion. In particular, we see the relation between the minimum sufficient set problem and the minimum vertex cover problem.

First we give some definitions. Define a vertex v to be a *branching node* if its degree in $s(G)$ is greater than 2. Analogously, define a *straight node* to be a node whose degree in $s(G)$ is less than or equal to 2. We will assume that $s(G)$ contains at least one branching node, since otherwise $s(G)$ is either a path or a cycle, and the sufficient sets of the two structures have been considered by Wilfong and Winkler[3].

Wilfong and Winkler[3] have also developed some basic properties of converters in a bi-directed graph. The following theorem is proved in Ref.[3].

Theorem 3[3]. Let G be a bi-directed graph. A set S of nodes is sufficient for G if and only if each component of $G_s(S)$ is a spider.

In Ref.[4], a sufficient set is said to be *canonical* if it contains only branching nodes. A straight node never needs to be included in an optimal sufficient set because any path passing through a straight node does not necessitate a converter.

Proposition 1. Let G be a bi-directed graph. If S is a sufficient set for G, then there exists another sufficient set S' for G such that $|S'| \le |S|$ and S' (as a subset of vertices in $G_s(S)$) does not contain any straight node.

In Ref.[4], Kleinberg and Kumar constructed a graph $H=(V_H,E_H)$ as follows. V_H consists of all branching nodes in $s(G)$. For $u,v \in V_H$, (u,v) is an edge in E_H if and only if there exists a path in $s(G)$ between u and v such that all internal nodes in the path are straight nodes. Note that H may have self-loops, which we retain as part of the graph. The following theorem is proved in Ref.[4] by Theorem 3.

Theorem 4[4]. Let S be a canonical sufficient set, and consider S as a subset of V_H. Then S is a vertex cover of H. Conversely, every vertex cover of H is also a sufficient set of G.

By Theorem 4, we can get the following conclusion:

Corollary 1. If there is a polynomial-time c-approximation for the minimum vertex cover problem, then there is a polynomial-time c-approximation for the minimum sufficient set.

Proof. Let S denote the set output by a polynomial-time c-approximation algorithm for the minimum vertex cover of the graph H. Let C^* denote the minimum vertex cover of H. Thus, we get $|S| \le c|C^*|$. By Theorem 4, both S

and C^* are sufficient sets of G. Let S^* denote the minimum sufficient set of G, so $|S^*|\le|C^*|$. Therefore, we get $|S|\le c|S^*|$, i.e., S is also a sufficient set whose size within a factor c of minimum. □

From Corollary 1, we see that the minimum sufficient set problem in general graphs can be transformed into minimum vertex cover problem. We know greedy algorithm is a 2-approximation algorithm. We present Algorithm II by revising the traditional greedy algorithm.

Algorithm 2.

Input: H, $S\leftarrow\varnothing$;
while edges remain {
  while vertices of degree 1 remain {
    select a vertex v of degree 1;
    add its adjacent vertex u to S;
    remove u, v and all the edges incident to them;
  }
  select an arbitrary edge (i, j) with its each end of degree ≤ 2;
  add i and j to S;
  remove the edge (i, j) and all the edges adjacent to (i, j);
}
Output: S.

We can get a PTAS (*polynomial time approximation scheme*) algorithm of sufficient set in planar bi-directed networks by applying the Lipton-Tarjan planar separator theorem[5] for vertex cover in planar graphs. We can also use an efficient polynomial algorithm for maximum matching in bipartite graphs to get a minimum sufficient set in bipartite-shaped networks. Please read Refs.[6,7] for the detailed algorithms.

Similar to the proof of Corollary 1, we can show the following resolution by applying Theorem 4.

Corollary 2. If there is a polynomial-time c-approximation for the minimum sufficient set problem, then there is a polynomial-time c-approximation for the minimum vertex cover problem.

4 Remarks

Our work proposes some practical and efficient algorithms for a natural class of optical networks. These algorithms are natural and it would be interesting to empirically test whether these perform well in reality.

It has to be mentioned that the paper draws a lot of techniques from Ref.[4].

The existence of a polynomial-time c-approximation for the minimum vertex cover with $c<2$ remains a longstanding open question. We can consider the interesting question.

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全光双向网络中的波长转换

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摘要: 在许多光学路由中,对于给定一组通讯路的集合,必须对有公共边的路安排相同的波长.为了充分利用光学的带宽,目的是安排尽量少的波长数.但有时候也考虑使用波长转换器.如果一个顶点安装转换器,任何经过这个顶点的路都可以改变其波长.因此在某些顶点安装波长转换器后可以将波长的数目减少到一个拥塞界,因此,Wilfong 和 Winkler 定义了一个顶点集 *S*,在 *S* 上安装转换器后,任何路集都可以分配数目等于拥塞界的波长,这样的集合 *S* 被称为充分集.研究在双向网络中的最小充分集问题,并把他转化为最小顶点覆盖问题.对此问题给出几个算法.

关键词: 近似算法; WDM 网络;波长转换;顶点覆盖;充分集
中图法分类号: TP393 文献标识码: A