

(3) 先验信息融合性不强

现有的矩阵补全理论往往忽略了数据固有的先验信息,这类先验信息通常包括目标矩阵的先验稀疏信息、应用问题的先验特征信息和结构信息等.例如:Cabral 等人在文献[5]中利用矩阵补全的方法解决多标记图像分类问题时,忽略了图像与图像、标记与标记之间的先验相似性关联信息;Mardani 等人在文献[6]利用矩阵补全方法监测网络异常流量时,忽略了网络流量的先验特征和结构信息;Natarajan 等人在文献[87]中将矩阵补全算法应用于基因-疾病预测时,忽略了基因-疾病关联矩阵的先验稀疏信息.

(4) 算法扩散性及计算效率低下

现有矩阵补全算法的可扩散性和计算效率也已成为其大规模应用的瓶颈.针对这个问题,研究者已经进行过一些探索.针对核范数松弛类矩阵补全算法所涉及的奇异值分解问题,普遍采用的方法是调用 PROPACK 软件包进行部分奇异值分解,从而有效降低奇异值分解的时间复杂度,一定程度上加速了算法收敛速度,但仍然无法适用于大规模矩阵补全问题.为此,Teflioudi 等人在文献[88]中基于 MapReduce 并行编程模型设计了一类适用于集群平台的并行交替最小均方算法,Recht 等人在文献[75]中基于随机投影增量梯度方法设计了一类矩阵补全并行算法,Mosabbeh 等人在文献[89]中基于交替方向乘法设计了一类分布式矩阵补全并行算法并实际应用于大规模多标记分类,而 Makari 等人则在文献[30]中基于随机梯度下降方法设计了一类既能运行在共享存储又能运行在分布式存储平台上的并行算法.然而,所有这些算法一方面局限于解决直传型矩阵补全问题,缺乏归纳推广性能;另一方面,基本都是基于 MapReduce 并行编程模型设计,仅适合运行于 Hadoop 平台,而且由于矩阵补全算法需要大量的迭代计算,使用 MapReduce 编程模型效率显得极为低下,仍然难以满足很多复杂的大数据处理需求.

4.2 未来研究方向

矩阵补全技术研究方兴未艾,在模型、算法及其应用方面仍需要做很多工作.下面分别从这几个方面对其未来研究方向进行探讨.

(1) 模型方面

现有的矩阵补全模型往往局限于数据表示信息的重建,重建的目的是获得最佳描述特征,未能有效利用数据的判别信息.例如:在多标记学习任务中经常遇到基于不完全样本特征信息的多标记分类问题,对于此类问题,我们实际上并不关心补全后的样本数据和真实的样本数据是否在数据表示上完全一致,我们真正关心的是这些补全后的样本数据能否被正确分类,因此,如果在对这些样本特征信息进行预测和补全时能够融入监督或半监督的类别信息,那么将有可能提高后续分类任务的分类性能.因此,研究如何在问题建模时融入问题的判别信息将具有很好的理论与现实意义,极有可能成为矩阵补全技术研究的新热点.此外,矩阵补全是压缩感知从一维向量空间向二维矩阵空间的拓展,而张量补全^[93]则是矩阵补全从二维矩阵空间向多维张量空间的拓展.与压缩感知和矩阵补全技术相比,张量补全技术研究起步较晚,张量补全模型多样性的缺乏阻碍了其在实际应用中的推广,因此可以预见,张量补全技术的研究将是一个值得持续关注的热点.

(2) 算法方面

现有的矩阵补全算法大多属于集中式串行处理算法,受制于单机计算效率和内存空间限制,算法的扩散性和计算效率低下,难以应用于大规模问题.对现有算法进行分布式并行扩展,应该是一种可行的解决思路.典型的分布式并行编程模型诸如 MPI,Hadoop 和 Spark 各有其优缺点,其中,源于加州伯克利大学的 Spark 是近年来大数据处理平台的新锐代表.Spark 已经在批处理、流计算、机器学习、图计算、SQL 查询等一系列领域得到广泛应用,并随着愈发活跃的开发者社区以及 Twitter,Adobe,Intel,Amazon,Redhat 等公司的加入而渐成气候.相比于 Hadoop,Spark 是内存计算框架,拥有 Hadoop MapReduce 所具有的优点;但不同于 Hadoop,Job 中间输出和结果可以保存在内存中,从而不再需要读写 HDFS,因此尤其适用于需要多次迭代计算的矩阵补全模型求解.而且从数值优化上看,Spark MLlib 已成功实现了梯度下降、牛顿法、ADMM 等经典优化算法.因此,Spark 有望成为矩阵补全分布式并行扩展的首选编程模型.

(3) 应用方面

现有的矩阵补全技术虽然在诸多领域得到了广泛应用,但是大多基于一些严格的假设,实际上这些假设并不一定符合实际,因此,如何放宽矩阵补全应用中的理想假设使其更加契合问题的本源,将是值得关注的研究方向.例如在多标记图像分类中,往往假设图像标记和图像特征之间满足线性映射关系,而实际情况是更可能满足非线性映射关系.再如,在推荐系统的用户-评分预测中,通常假设用户偏好在某一个时间段内是一成不变的,只在跨时间段间发生变化,这种假设显然也是过于严苛的.针对这两种理想假设的不足,最近已有学者分别提出了非线性矩阵补全^[91]和动态矩阵补全模型^[92],模型性能有了显著提高.此外,矩阵补全模型的噪声容错性能也是一个影响矩阵补全应用推广的普适性问题,现有的矩阵补全模型在噪声类型先验假设的前提下取得了较好的研究成果,但是不可预知的复杂噪声背景下的矩阵补全技术研究才刚刚起步.其中,西安交通大学徐宗本院士和孟德宇博士团队在低维子空间学习模型噪声容错性领域的研究成果为此提供了一个很好的思路,他们在 ICCV 2015 会议上撰文指出^[93]:如果采用混合指数幂分布(mixture of exponential power)函数来拟合不可预知的复合噪声类型,将取得令人鼓舞的效果.其理论依据是,混合指数幂分布函数理论上能逼近任意分布.虽然该方法目前尚存在算法扩散性和初值敏感性问题,但其无疑为机器学习领域众多学习模型的噪声容错性问题提供了一种很好的解决思路.

5 本文总结

作为稀疏学习理论的重要组成部分,衍生于压缩感知的矩阵补全技术近年来在机器学习领域获得了广泛关注,短短几年,从模型、算法到应用方面均得到了快速发展,取得了诸多研究成果.本文首先从秩函数松弛的角度综述了 4 类不同的矩阵补全模型,旨在为相关应用领域的矩阵补全问题建模提供参考;然后从独立于问题模型的角度综述了适用于矩阵补全模型统一框架的的常用优化算法,其目的在于从本质上加深对矩阵补全模型优化技巧的理解,从而有利于面向应用问题的矩阵补全新模型的优化求解;最后指出了现有矩阵补全模型在噪声容错性、归纳推广性和先验信息融合性方面存在的不足;同时,本文也关注到现有的矩阵补全算法大多受制于单机计算效率和内存空间限制,导致算法扩散性和计算效率低下,难以应用于大规模问题求解的问题.可以想见,如果上述这些问题能够得到较好的解决,矩阵补全技术必将迎来更加广阔的应用前景.

References:

- [1] Donoho DL. Compressed sensing. *IEEE Trans. on Information Theory*, 2006,52(4):1289–1306. [doi: 10.1109/TIT.2006.871582]
- [2] Candès EJ, Romberg J, Tao T. Robust uncertainty principles: Exact signal reconstruction from highly incomplete frequency information. *IEEE Trans. on Information Theory*, 2006,52(2):489–509. [doi: 10.1109/TIT.2005.862083]
- [3] Candès EJ, Recht B. Exact matrix completion via convex optimization. *Foundations of Computational Mathematics*, 2009,9(6): 717–772. [doi: 10.1007/s10208-009-9045-5]
- [4] Li W, Zhao L, Lin ZJ, Xu DQ, Lu DM. Non-Local image inpainting using low-rank matrix completion. *Computer Graphics Forum*, 2015,34(6):111–122. [doi: 10.1111/cgf.12521]
- [5] Cabral R, De la Torre F, Costeira JP, Bernardino A. Matrix completion for weakly-supervised multi-label image classification. *IEEE Trans. on Pattern Analysis and Machine Intelligence*, 2015,37(1):121–135. [doi: 10.1109/TPAMI.2014.2343234]
- [6] Mardani M, Giannakis GB. Estimating traffic and anomaly maps via network tomography. *IEEE/ACM Trans. on Networking*, 2016, 24(5):1533–1547. [doi: 10.1109/TNET.2015.2417809]
- [7] Yi KF, Wan JW, Yao L, Bao TY. Partial matrix completion algorithm for efficient data gathering in wireless sensor networks. *IEEE Communications Letters*, 2015,19(1):54–57. [doi: 10.1109/LCOMM.2014.2371998]
- [8] Otazo R, Candès E, Sodickson DK. Low-Rank plus sparse matrix decomposition for accelerated dynamic MRI with separation of background and dynamic components. *Magnetic Resonance in Medicine*, 2015,73(3):1125–1136. [doi: 10.1002/mrm.25240]
- [9] Taghizadeh MJ, Parhizkar R, Garner PN, Bourlard H, Asaei A. Ad hoc microphone array calibration: Euclidean distance matrix completion algorithm and theoretical guarantees. *Signal Processing*, 2015,107:123–140. [doi: 10.1016/j.sigpro.2014.07.016]
- [10] Hsieh CJ, Natarajan N, Dhillon IS. PU learning for matrix completion. In: *Proc. of the Int'l Conf. on Machine Learning*. 2015.

- [11] Adeli-Mosabbe E, Fathy M. Non-Negative matrix completion for action detection. *Image and Vision Computing*, 2015,39(1): 38–51. [doi: 10.1016/j.imavis.2015.04.006]
- [12] Yi J, Zhang L, Jin R, Qian Q, Jain AK. Semi-Supervised clustering by input pattern assisted pairwise similarity matrix completion. In: *Proc. of the Int'l Conf. on Machine Learning*. 2013.
- [13] Gogn A, Majumdar A. Matrix completion incorporating auxiliary information for recommender system design. *Expert System with Applications*, 2015,42(12):5789–5799. [doi: 10.1016/j.eswa.2015.04.012]
- [14] Candes EJ, Eldar YC, Strohmer T, Vladislav H. Phase retrieval via matrix completion. *SIAM Review*, 2015,57(2):225–251. [doi: 10.1137/151005099]
- [15] Kumar R, Silva CD, Akalin O, Aravkin AY, Mansour H; Recht B; Herrmann FJ. Efficient matrix completion for seismic data reconstruction. *Geophysics*, 2015, 80(5):97–114. [doi: 10.1190/geo2014-0369.1]
- [16] Zhao Z, Zhang LJ, He XF, Ng W. Expert finding for question answering via graph regularized matrix completion, *IEEE Trans. on Knowledge and Data Engineering*, 2015,27(4):993–1004. [doi: 10.1109/TKDE.2014.2356461]
- [17] Xiao F, Sha CH, Chen L, Sun LJ, Wang RC. Noise-Tolerant localization from incomplete range measurements for wireless sensor networks. In: *Proc. of the IEEE Int'l Conf. on Computer Communications*. 2015. 2794–2802. [doi: 10.1109/INFOCOM.2015.7218672]
- [18] Chen L, Yang G, Chen ZY, Xiao F, Xu J. Web services QoS prediction via matrix completion with structural noise. *Journal on Communications*, 2015,36(6):49–59 (in Chinese with English abstract).
- [19] Chen L, Yang G, Chen ZY, Xiao F, Shi JY. Correlation consistency constrained matrix completion for Web service tag refinement. *Neural Computing and Applications*, 2015,26(1):101–110. [doi: 10.1007/s00521-014-1704-z]
- [20] Bishop WE, Yu BM. Deterministic symmetric positive semidefinite matrix completion. In: *Proc. of the Advances in Neural Information Processing Systems*. 2014.
- [21] Li XD. Compressed sensing and matrix completion with constant proportion of corruptions. *Constructive Approximation*, 2013,37: 73–99. [doi: 10.1007/s00365-012-9176-9]
- [22] Candes EJ, Plan Y. Matrix completion with noise. *Proc. of the IEEE*, 2010,98(6):925–936. [doi: 10.1109/JPROC.2009.2035722]
- [23] Mardani M, Mateos G, Giannakis GB. Subspace learning and imputation for streaming big data matrices and tensors. *IEEE Trans. on Signal Processing*, 2015,63(10):2663–2677. [doi: 10.1109/TSP.2015.2417491]
- [24] Wang Z, Lai J, Lu Z S, Ye JP. Orthogonal rank-one matrix pursuit for low rank matrix completion *SIAM Journal on Scientific Computing*, 2015,37(1):488–514. [doi: 10.1137/130934271]
- [25] Ma S, Goldfarb D, Chen L. Fixed point and Bregman iterative methods for matrix rank minimization. *Mathematics Programming*, 2011,128(1):321–353. [doi: 10.1007/s10107-009-0306-5]
- [26] Xu YY, Yin WT, Wen ZW, Zhang Y. An alternating direction algorithm for matrix completion with nonnegative factors. *Frontiers of Mathematics in China*, 2012,7(2):365–384. [doi: 10.1007/s11464-012-0194-5]
- [27] Mohan K, Fazel M. Iterative reweighted algorithms for matrix rank minimization. *Journal of Machine Learning Research*, 2013,13: 3253–3285.
- [28] Mackey L, Talwalkar A, Jordan M. Distributed matrix completion and robust factorization. *Journal of Machine Learning Research*, 2015,16(1):913–960.
- [29] Candes EJ, Tao T. The power of convex relaxation: Near-optimal matrix completion. *IEEE Trans. on Information Theory*, 2009, 56(5):2053–2080. [doi: 10.1109/TIT.2010.2044061]
- [30] Makari F, Teflioudi C, Gemulla R, Haas P, Sismanis Y. Shared-Memory and shared-nothing stochastic gradient descent algorithms for matrix completion. *Knowledge and Information Systems*, 2015,42(3):493–523. [doi: 10.1007/s10115-013-0718-7]
- [31] Nie FP, Wang H, Huang H, Ding C. Joint Schatten p -norm and L_p -norm robust matrix completion for missing value recovery. *Knowledge and Information Systems*, 2015,42(3):525–544. [doi: 10.1007/s10115-013-0713-z]
- [32] Lu CY, Tang JH, Yan SC, Lin Z. Generalized nonconvex non-smooth low-rank minimization. In: *Proc. of the IEEE Conf. on Computer Vision and Pattern Recognition*. 2014. 4130–4137. [doi: 10.1109/CVPR.2014.526]
- [33] Zhang LJ, Yang T, Jin R, Zhou ZH. Stochastic proximal gradient descent for nuclear norm regularization. 2015. <http://arxiv.org/abs/1511.01664> CoRR abs/1511.01664

- [34] Boumal N, Absil P. RTRMC: A riemannian trust region method for matrix completion. In: Proc. of the Annual Conf. on Neural Information Processing Systems. 2011.
- [35] Ghasemi H, Malek-Mohammadi M, Babaie-Zadeh M. SRF: Matrix completion based on smoothed rank function. In: Proc. of the IEEE Int'l Conf. on Acoustics, Speech and Signal Processing. 2011. 3672–3675. [doi: 10.1109/ICASSP.2011.5947147]
- [36] Winlaw M, Hynes M, Caterini A, Sterck H. Algorithmic acceleration of parallel ALS for collaborative filtering; speeding up distributed big data recommendation in Spark. In: Proc. of the IEEE Int'l Conf. on Parallel and Distributed Systems. 2015. 682–691. [doi: 10.1109/ICPADS.2015.91]
- [37] Sun RY, Luo ZQ. Guaranteed matrix completion via non-convex factorization. In: Proc. of the Annual Symp. on Foundations of Computer Science. 2015. [doi: 10.1109/FOCS.2015.25]
- [38] Lin ZC. Rank minimization: Theory, algorithms and applications. In: Zhang CS, Zhou ZH, eds. Proc. of the Machine Learning and Their Applications. Beijing: Tsinghua University Press, 2013. 143–164 (in Chinese with English abstract).
- [39] Peng YG, Suo JL, Dai QH, Xu LW. From compressed sensing to low-rank matrix recovery: Theory and applications. *Acta Automatica Sinica*, 2013,39(7):981–994 (in Chinese with English abstract). [doi: 10.1016/S1874-1029(13)60063-4]
- [40] Chen CH, He BS, Yuan XM. Matrix completion via an alternating direction method. *IMA Journal of Numerical Analysis*, 2012, 32(1):227–245. [doi: 10.1093/imanum/drq039]
- [41] Wen SW, Xu FF, Wen ZW, Chen L. Robust linear optimization under matrix completion. *Science in China: Mathematics*, 2014,57: 699–710. [doi: 10.1007/s11425-013-4697-7]
- [42] Jiao LC, Zhao J, Yang SY, Liu F, Xie W. Research advances on sparse cognitive learning, computing and recognition. *Chinese Journal of Computers*, 2016,39(4):835–852 (in Chinese with English abstract).
- [43] Hu Y, Zhang DB, Ye JP, Li X, He XF. Fast and accurate matrix completion via truncated nuclear norm regularization. *IEEE Trans. on Pattern Analysis and Machine Intelligence*, 2013,35(9):2117–2130. [doi: 10.1109/TPAMI.2012.271]
- [44] Chen L, Yang G, Chen ZY, Xiao F, Chen SC. Linearized bregman iteration algorithm for matrix completion with structural noise. *Chinese Journal of Computers*, 2015,38(7):1357–1371 (in Chinese with English abstract).
- [45] Tibshirani R. Regression shrinkage and selection via the lasso. *Journal of the Royal Statistical Society*, 1996,58(1):267–288.
- [46] Fazel M. Matrix rank minimization with applications [Ph.D. Thesis]. Stanford University, 2002.
- [47] Toh KC, Yun SW. An accelerated proximal gradient algorithm for nuclear norm regularized least squares problems. *Pacific Journal of Optimization*, 2010,6(3):615–640.
- [48] Cai JF, Candes EJ, Shen Z. A singular value thresholding algorithm for matrix completion, *SIAM Journal of Optimization*, 2010, 20(4):1956–1982. [doi: 10.1137/080738970]
- [49] Lin ZC, Chen MM, Wu LQ, Ma Y. The augmented Lagrange multiplier method for exact recovery of corrupted low-rank matrices. Technical Report, UILU-ENG-09-2214, UIUC University, 2010.
- [50] Wen ZW, Yin WT, Zhang Y. Solving a low rank factorization model for matrix completion by a nonlinear successive over-relaxation algorithm. *Mathematic Programming Computation*. 2012,4:333–361. [doi: 10.1007/s12532-012-0044-1]
- [51] Liu YY, Jiao LC, Shang FH. A fast tri-factorization method for low-rank matrix recovery and completion. *Pattern Recognition*, 2013,46(1):163–173. [doi: 10.1016/j.patcog.2012.07.003]
- [52] Srebro N, Shraibman A. Rank, trace-norm and max-norm. In: Proc. of the Annual Conf. on Learning Theory. 2005. 545–560. [doi: 10.1007/11503415_37]
- [53] Mardani M, Mateos G, Giannakis GB. Rank minimization for subspace tracking from incomplete data. In: Proc. of the IEEE Int'l Conf. on Acoustics, Speech and Signal Processing. 2013. 5681–5685. [doi: 10.1109/ICASSP.2013.6638752]
- [54] Geng J, Wang LS, Wang YF. A non-convex algorithm framework based on DC programming and DCA for matrix completion. *Numerical Algorithms*, 2015,68:903–921. [doi: 10.1007/s11075-014-9876-2]
- [55] Kang Z, Peng C, Cheng J, Cheng Q. LogDet rank minimization with application to subspace clustering. *Computational Intelligence and Neuroscience*, 2015, Article ID 824289. [doi: 10.1155/2015/824289]
- [56] Honerkamp J, Weese J. Tikhonovs regularization method for ill-posed problems. *Continuum Mechanics and Thermodynamics*, 1990,2(1):17–30. [doi: 10.1007/BF01170953]

- [57] Chen Z, Haykin S. On different facets of regularization theory. *Neural Computation*, 2002,14(12):2791–2846. [doi: 10.1162/089976602760805296]
- [58] Combettes PL, Wajs VR. Signal recovery by proximal forward-backward splitting. *Multiscale Modeling and Simulation*, 2005,4(4):1168–1200. [doi: 10.1137/050626090]
- [59] Nemirovski A. Efficient methods in convex programming. *Lecture Notes*, 1995,23(3):24–39.
- [60] Beck A, Teboulle M. A fast iterative shrinkage-thresholding algorithm for linear inverse problems. *SIAM Journal on Imaging Sciences*, 2009,2(1):183–202. [doi: 10.1137/080716542]
- [61] Nesterov Y. A method of solving a convex programming problem with convergence rate $O(k^{-2})$. *Soviet Mathematics Doklady*, 1983,27(2):372–376.
- [62] Nesterov Y. Efficiency of coordinate descent methods on huge-scale optimization problems. *SIAM Journal on Optimization*, 2012,22:341–362. [doi: 10.1137/100802001]
- [63] Beck A, Tetruashvili L. On the convergence of block coordinate descent methods. *SIAM Journal on Optimization*, 2013,23(4):2037–2060. [doi: 10.1137/120887679]
- [64] Bregman L. The relaxation method of finding the common points of convex sets and its application to the solution of problems in convex programming. *Computational Mathematics and Mathematical Physics*, 1967,7(3):200–217. [doi: 10.1016/0041-5553(67)90040-7]
- [65] Osher S, Burger M, Goldfarb M. An iterative regularization method for total variation-based image restoration. *Multiscale Modeling and Simulation*, 2005,4(2):460–489. [doi: 10.1137/040605412]
- [66] Yin WT, Osher S, Goldfarb D. Bregman iterative algorithms for L_1 -minimization with applications to compressed sensing. *SIAM Journal on Imaging Sciences*, 2008,1(1):143–168. [doi: 10.1137/070703983]
- [67] Cai JF, Osher S, Shen Z. Linearized Bregman iteration for frame-based image deblurring. *SIAM Journal on Imaging Sciences*, 2009,2(2):226–252. [doi: 10.1137/080733371]
- [68] Boyd S, Parikh N, Chu E. Distributed optimization and statistical learning via the alternating direction method of multipliers. *Foundations and Trends® in Machine Learning*, 2011,3(1):1–122. [doi: 10.1561/2200000016]
- [69] Douglas J, Rachford HH. On the numerical solution of heat conduction problems in two and three space variables. *Trans. of the American Mathematical Society*, 1956,82:421–439. [doi: 10.1090/S0002-9947-1956-0084194-4]
- [70] Gabay D, Mercier B. A dual algorithm for the solution of nonlinear variational problems via finite element approximations. *Computers and Mathematics with Applications*, 1976,2:17–40. [doi: 10.1016/0898-1221(76)90003-1]
- [71] Zhang RL, Kwok J. Asynchronous distributed ADMM for consensus optimization. In: *Proc. of the Int'l Conf. on Machine Learning*. 2014.
- [72] Zhang WL, Kwok J. Fast stochastic alternating direction method of multipliers. In: *Proc. of the Int'l Conf. on Machine Learning*. 2014.
- [73] He BS, Yuan XM. On the $O(1/m)$ convergence rate of the Douglas-Rachford alternating direction method. *SIAM Journal of Numerical Analysis*. 2012,50(2):700–709. [doi: 10.1137/110836936]
- [74] He BS, Tao M, Yuan XM. Alternating direction method with Gaussian back substitution for separable convex programming. *SIAM Journal on Optimization*, 2012,22(2):313–340. [doi: 10.1137/110822347]
- [75] Lin ZC, Liu RS, Li H. Linearized alternating direction method with parallel splitting and adaptive penalty for separable convex programs in machine learning. *Machine Learning*, 2015,99(2):287–325. [doi: 10.1007/s10994-014-5469-5]
- [76] Nemirovski A, Juditsky A, Lan G, Shapiro A. Robust stochastic approximation approach to stochastic programming. *SIAM Journal on Optimization*, 2009,19(4):1574–1609. [doi: 10.1137/070704277]
- [77] Recht B, Re C. Parallel stochastic gradient algorithms for large-scale matrix completion. *Mathematical Programming Computation*, 2013,5(2):201–226. [doi: 10.1007/s12532-013-0053-8]
- [78] Liu J, Wright SJ, Re C, Bittorf V, Sridhar S. An asynchronous parallel stochastic coordinate descent algorithm. *Journal of Machine Learning Research*, 2015,16(1):285–322.
- [79] Azadi S, Sra S. Towards an optimal stochastic alternating direction method of multipliers. In: *Proc. of the Int'l Conf. on Machine Learning*. 2014.

- [80] Zhang LJ, Yang T, Yi J, Jin R, Zhou ZH. Stochastic optimization for kernel PCA. In: Proc. of the AAAI Conf. on Artificial Intelligence. 2016. 2316–2322.
- [81] Zhang ZH. The singular value decomposition, applications and beyond. arXiv: 1510.08532v1, <http://arxiv.org/abs/1510.08532>
- [82] Tao M, He BS, Yuan XM. Recovering low-rank and sparse components of matrices from incomplete and noisy observations. SIAM Journal on Optimization, 2011,21(1):57–81. [doi: 10.1137/100781894]
- [83] Yan M, Yang Y, Osher S. Exact low-rank matrix completion from sparsely corrupted entries via adaptive outlier pursuit. Journal of Scientific Computing, 2013,56(3):433–449. [doi: 10.1007/s10915-013-9682-3]
- [84] Chen YD, Jalali A, Sanghavi S, Caramanis C. Low-Rank matrix recovery from errors and erasures. IEEE Trans. on Information Theory, 2013,59(7):4324–4337. [doi: 10.1109/TIT.2013.2249572]
- [85] Chen YD, Xu H, Caramanis C, Sanghavi S. Robust matrix completion and corrupted columns. In: Proc. of the Int'l Conf. on Machine Learning. 2011.
- [86] Xu M, Jin R, Zhou ZH. Speedup matrix completion with side information: Application to multi-label learning. In: Proc. of the Advances in Neural Information Processing Systems. 2013.
- [87] Natarajan N, Dhillon IS. Matrix completion for predicting gene-disease associations. Bioinformatics, 2014,30(12):60–68. [doi: 10.1093/bioinformatics/btu269]
- [88] Teflioudi C, Makari F, Gemulla R. Distributed matrix completion. In: Proc. of the IEEE Int'l Conf. on Data Mining. 2012. 655–664. [doi: 10.1109/ICDM.2012.120]
- [89] Mosabbeh EA, Fathy M. Distributed matrix completion for large-scale multi-label classification. Intelligent Data Analysis, 2014,18: 1137–1151.
- [90] Liu J, Musialski P, Wonka P, Ye JP. Tensor completion for estimating missing values in visual data. IEEE Trans. on Pattern Analysis and Machine Intelligence, 2013,35(1):208–220. [doi: 10.1109/TPAMI.2012.39]
- [91] Alameda-Pineda X, Yan Y, Ricci E, Sebe N. Recognizing emotions from abstract paintings using non-linear matrix completion. In: Proc. of the IEEE Conf. on Computer Vision and Pattern Recognition. 2016. 5240–5248. [doi: 10.1109/CVPR.2016.566]
- [92] Xu LB, Davenport MA. Dynamic matrix recovery from incomplete observations under an exact low-rank constraint. In: Proc. of the Advances in Neural Information Processing Systems. 2016.
- [93] Cao XR, Chen Y, Zhao Q, Meng DY, Wang Y, Wang D, Xu ZB. Low-Rank matrix factorization under general mixture noise distributions. In: Proc. of the Int'l Conf. on Computer Vision. 2015. [doi: 10.1109/ICCV.2015.175]

附中中文参考文献:

- [18] 陈蕾,杨庚,陈正宇,肖甫,许建.基于结构化噪声矩阵补全的 Web 服务 QoS 预测.通信学报,2015,36(6):49–59.
- [38] 林宙辰.秩极小化:理论、算法与应用.见:张长水,周志华,编.机器学习及其应用会议论文集.北京:清华大学出版社,2013.143–164.
- [39] 彭义刚,索津莉,戴琼海,徐文立.从压缩传感到低秩矩阵恢复:理论与应用.自动化学报,2013,39(7):981–994. [doi: 10.1016/S1874-1029(13)60063-4]
- [42] 焦李成,赵进,杨淑媛,刘芳,谢雯.稀疏认知学习、计算与识别的研究进展.计算机学报,2016,39(4):835–852.
- [44] 陈蕾,杨庚,陈正宇,肖甫,陈松灿.基于线性 Bregman 迭代的结构化噪声矩阵补全算法.计算机学报,2015,38(7):1357–1371.



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