计算 $\mathcal{R}_i=\mathcal{R}_i-\bigcup_{j=0}^{i-1}\mathcal{R}_j$; //去掉$\mathcal{R}_i$中与前面集合重复点集;

}

//设$\mathcal{R}_n$={[$-a_n$,$-\Delta f$,$-a_n$],[a_n,$a_n+\Delta f$]}

步骤 4:计算 $sum1=\sum_{i=0}^{n}\exp(-i\varepsilon)\mu(\mathcal{R}_i)$ 及 $sum2=2\Delta f*\exp(-\varepsilon(n+1))/(1-\exp(-\varepsilon))$;

步骤 5:计算$\alpha=sum1+sum2$;

步骤 6:设置 $p_i(r)=\exp(-i\varepsilon)/\alpha$,其中,$r\in\mathcal{R}_i$,$i\in\mathbb{N}$;

步骤 7:Return $\{(p_i(r),\mathcal{R}_i):i\in\mathbb{N}\}$.

算法 1 需要下面的集合序列收敛性质:

定义 **2**. 设$\mathcal{R}=\mathbb{R}$.对数据集 $x\in\mathcal{D}$,称集合序列$\{\mathcal{R}_i:i\in\mathbb{N}\}$在第 $n\in\mathbb{N}$步收敛,如果存在 $a_n\in\mathcal{R}$,有:

$$\mathcal{R}_n=\pm(a_n+[0,\Delta f])\text{且}\mathcal{R}_{n+1}=\pm(a_n+[\Delta f,2\Delta f]).$$

我们的实验设计如下:首先,计算算法 1 生成的密度函数的数学期望值 $mean=\sum_{i=0}^{\infty}p_i\times\int_{r\in\mathcal{R}_i}r\mu(\mathrm{d}r)$; 然后,与 Laplace 机制[27]的期望值$\Delta f/\varepsilon$及 Staircase 机制[18]的期望值$\Delta f\times\exp(\varepsilon/2)/(\exp(\varepsilon)-1)$进行比较,越小的期望值代表噪声复杂度越低.由于发现 Staircase 机制的期望值比 Laplace 机制的期望值小,因此,本实验只比较 *mean* 与 Staircase 机制期望值$\Delta f\times\exp(\varepsilon/2)/(\exp(\varepsilon)-1)$的大小,即 $rate=mean\times(\exp(\varepsilon)-1)/(\Delta f\times\exp(\varepsilon/2))$.*rate* 值小于 1,代表算法 1 对应机制优于 Staircase 机制;反之,则差于 Staircase 机制.

我们构造两组实验:第 1 组使用相同的敏感度及不同的函数邻居集测度,第 2 组使用不同的敏感度及相同的函数邻居集测度.第 1 组实验有 4 个不同的查询函数 f_1,f_2,f_3,f_4,其邻居集分别为$\mathcal{V}_1$=[0,1]∪[1000,1001],$\mathcal{V}_2$=[0,100]∪[1000,1001],$\mathcal{V}_3$=[0,500]∪[1000,1001],$\mathcal{V}_4$=[0,1001].4 个不同集合可以理解为 4 个不同单位的工资分布情况:$\mathcal{V}_1$代表了工资两级分化极其严重的单位;$\mathcal{V}_4$代表了该单位的工资虽然最高工资和最低工资差距很大,但是高中低档工资都可以出现;其他两个集合代表了折中的情形.实验结果如图 1 所示.

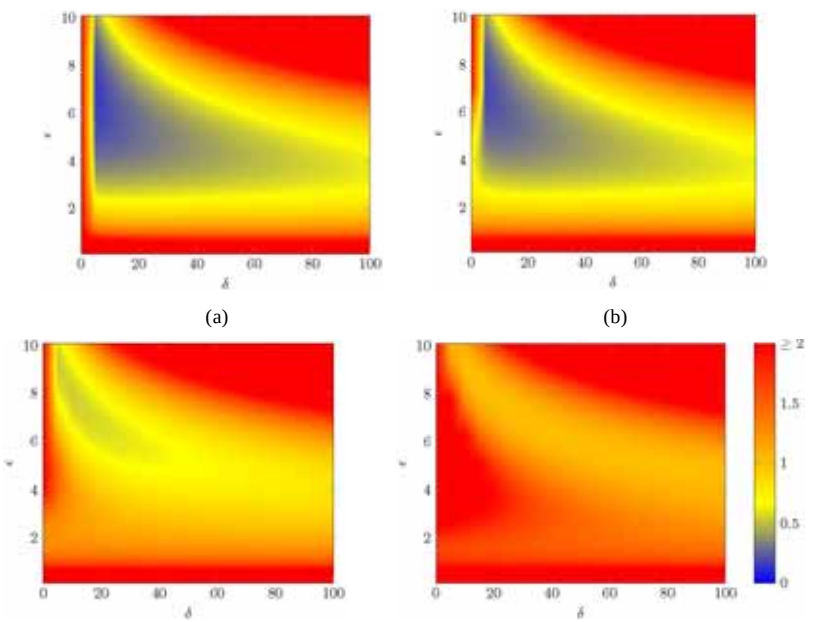
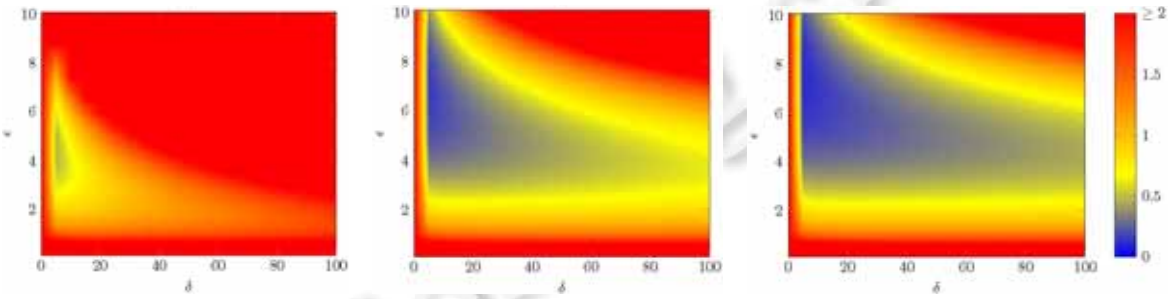


Fig.1

图 1

图 1 的 4 个子图分别代表了$\mathcal{V}_1,\mathcal{V}_2,\mathcal{V}_3,\mathcal{V}_4$的实现效果图.其中,横坐标表示$\delta$的取值,纵坐标表示$\varepsilon$的取值,坐标点$(\delta,\varepsilon)$对应的值表示相应的 *rate* 的值.从图 1 可以发现:随着邻居集测度$\mu(\mathcal{V}_i)$的增大,*rate* 的值相应增大,从而说明算法 1 中机制的优势是随$\mu(\mathcal{V}_i)$的增大而递减的.

第 2 组实验有 3 个查询函数,其邻居集分别为$\mathcal{V}_5$=[0,1]∪[100,101],$\mathcal{V}_6$=[0,1]∪[1000,1001],$\mathcal{V}_7$=[0,1]∪[2000,2001].显然,3 个函数的全局(及局部)敏感度分别为 101,1001,2001,但有相同的邻居集测度$\mu(\mathcal{V}_i)$=2.其实验结果如图 2 所示.



(a) (b) (c)

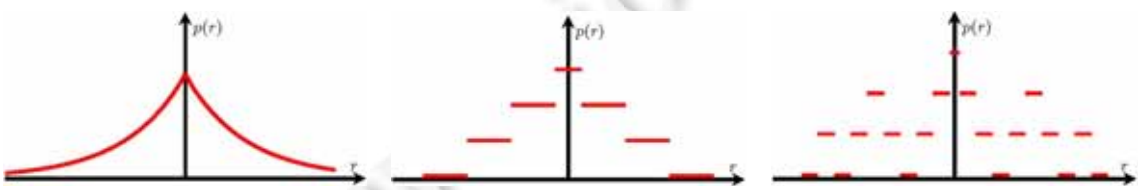
Fig.2

图 2

图 2 的 3 个子图分别代表了$\mathcal{V}_5,\mathcal{V}_6,\mathcal{V}_7$的实现效果图.与图 1 一样,每个子图中坐标点$(\delta,\varepsilon)$对应的值表示相应的 *rate* 的值.从图 2 可以发现:当敏感度增加的时候,算法 1 中机制的噪声复杂度要比 Staircase 机制的复杂度相对越来越小.也就是说:在函数邻居集的测度不变化的条件下,随着敏感度的增加,算法 1 中机制的噪声复杂度要比 Staircase 等敏感度方法的噪声复杂度越来越小.

综合图 1、图 2 的结果,我们可以得到如下的结论:若查询函数的邻居集$\mathcal{V}$的敏感度Δf与其测度$\mu(\mathcal{V})$的比率$\Delta f/\mu(\mathcal{V})$越来越大时,敏感度方法将添加越来越大的噪声.而本文的方法会极大地降低这个比率带来的噪声复杂度升高的问题,这也是本文的方法优于敏感度方法的最显著特征.

Laplace 机制、Staircase 机制及算法 1 中机制的密度函数如图 3 所示.图 3(c)的密度函数与前两个密度函数的显著区别是:密度函数并不是从中心向两边递减的,而是总体递减,但是局部有增的密度函数.这种密度函数更适合于$\Delta f/\mu(\mathcal{V})$很大的情形,因为其波浪状形态可以不受全局敏感度的限制而更适合于邻居集的(不同查询函数的)多变特性.



(a) Laplace 机制 (b) Staircase 机制 (c) 算法 1 中机制

Fig.3 The density functions of Laplace mechanism, Staircase mechanism and the mechanism in algorithm 1

图 3 Laplace 机制、Staircase 机制、算法 1 中机制的密度函数

算法 2 是算法 1 生成的密度函数的随机变量生成算法.

算法 **2**. 生成随机变量.

输入:敏感度Δf、迭代次数 n、$\{\mathcal{R}_i:i\in\{0,1,\ldots,n\}\}$、$a_n$、$\alpha=sum1+sum2$;

//设$\mathcal{R}_n=\{[-a_n,-\Delta f,-a_n],[a_n,a_n+\Delta f]\}$,即,算法从第 n 步后收敛;

输出:随机变量 X.

步骤 1:以 $\exp(-i\varepsilon)\mu(\mathcal{R}_i)/\alpha$的概率输出 $i,i\in\{0,1,\ldots,n\}$;以概率 $sum2/\alpha$输出 $n+1$;

步骤 2:If 步骤 1 输出值 $i\in\{0,1,\ldots,n\}$ then

在$\mathcal{R}_i$ 中均匀抽样出一个数 x;

Return x;

Else

从参数为 $1-\exp(-\varepsilon)$的几何分布中抽样出一个整数 j;

从$[-a_n-(j+1)\Delta f,-a_n-j\Delta f]\cup[a_n+j\Delta f,a_n+(j+1)\Delta f]$中均匀随机抽样出一个数 y;

Return y;

6.1 时间复杂度

算法 1 和算法 2 的时间复杂度由集合序列$\{\mathcal{R}_i:i\in\mathbb{N}\}$的收敛性决定.本节的例子中,该序列都在 2 200 步内达到了收敛.但是对一般线性查询函数,我们还没能有方法证明其一定在有限步内收敛.不过,我们相信这个结论是对的.

若$\{\mathcal{R}_i:i\in\mathbb{N}\}$在第 n 步收敛,且 $\max_{i\in\{0,1,\ldots,n\}}N_i=N$,其中,$N_i$ 是$\mathcal{R}_i$ 中区间的个数,则算法 1 的时间复杂度为 $O(n^2\times N^2)$,算法 2 的时间复杂度为 $O(n)$.

7 结 论

传统的观点认为,查询函数的(全局或局部)敏感度是查询函数噪声复杂度的(最主要)标志.本文的结论发现:敏感度其实只是查询函数的邻居集$\mathcal{V}^x$ 的一个极值特征,还有很多敏感度无法刻画的特征(如该集合中点的分布).本文的方法可根据$\mathcal{V}^x$ 中点的分布情形,构造适合于该分布的差分隐私机制,相应的密度函数类似于图 3(c)所示.与敏感度方法(如图 3(a)、图 3(b)所示)不同,本文机制的密度函数不是(向两边)全局递减的,而是以局部有起伏的方式(向两边)递减的,这种递减方式更具灵活性,更适合于$\mathcal{V}^x$ 中点的不规则分布情形.

第 6 节的实际例子也说明,本文的方法一般要比 Laplace 机制及 Staircase 机制更精确.但是本文的方法具有很高的时间复杂度,不适合于直接使用在大数据集上.如何通过本文的方法和结论构造低时间复杂度的非敏感度方法,将作为未来的一项工作.

另外,本文的很多分析方法和结论(如第 3 节、第 4 节中的内容都没有设定函数类型)也可对非线性查询问题[15,16,34−36]进行分析,因为非线性查询问题具有更加复杂的邻居集.至于其分析有效程度如何,还需要进一步探索.非线性查询问题将具有更加复杂的最优机制,不同数据集将对应形状各异的密度函数,对非线性函数的非敏感度机制研究将作为未来的另一项工作.

致谢 本文作者感谢匿名审稿专家对本文初稿提出的宝贵建议和意见,这些建议和意见对本文的完整性及易读性有很大的帮助.同时,这些建议和意见促使我们发现了初稿中的一个证明错误.

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武跟强(1980－),男,甘肃天水人,博士生,CCF 学生会员,主要研究领域为安全多方计算与隐私保护,差分隐私理论.

贺也平(1962－),男,博士,研究员,博士生导师,主要研究领域为系统安全,隐私保护.

夏娴瑶(1986－),女,博士生,CCF 学生会员,主要研究领域为隐私保护.