$$\rho_{i\to k}(c_i)=s(i,c_i)+s'(i,c_i')+\sum_{k':k'\neq k}\alpha_{i\leftarrow k'}(c_i)+[c_i\in neb(i,c_k')]\cdot\lambda_2\cdot I \tag{20}$$

从惩罚函数传递给潜在代表点的α-消息如下:

$$\alpha_{i\to k}(c_i)=\overbrace{\max_{j_1,\dots,j_{i-1},j_{i+1},\dots,j_N}\left[\Delta_k(j_1,\dots,j_{i-1},c_i,j_{i+1},\dots,j_N)+\sum_{i'}\rho_{i'\to k}(j_{i'})\right]}^{\text{best possible configuration satisfying }\Delta_k\text{ given }c_i}$$

$$=\begin{cases}\overbrace{[k\in neb(c_k')]\cdot\sum_{\substack{i':s(i',k)\leqslant maxval,\\ i'\neq k}}(\rho_{i'\to k}(k)+\lambda_2\cdot I)+\sum_{i'':i''\neq i',k}\max_{j'}\rho_{i''\to k}(j')}^{\text{best configuration with or without cluster }k}, & \text{for } c_i=k=i,\\ \overbrace{\sum_{i':i'\neq k}\max_{j':j'\neq k}\rho_{i'\to k}(j')}^{\text{best configuration with no cluster }k}, & \text{for } c_i\neq k=i,\\ \overbrace{\rho_{k\to k}(k)+[k\in neb(c_k')]\cdot\lambda_2\cdot I}^{k\text{ is an exemplar}}+\\ \overbrace{[k\in neb(c_k')]\cdot\sum_{\substack{i':s(i',k)\leqslant maxval,\\ i'\neq i,k}}(\rho_{i'\to k}(k)+\lambda_2\cdot I)+\sum_{i'':i\neq i',i,k}\max_{j'}\rho_{i''\to k}(j')}^{\text{best configuration of others}}, & \text{for } c_i=k\neq i\\ \max\left[\begin{array}{l}\overbrace{\max_{j':j'\neq k}\rho_{k\to k}(j')+\sum_{i':i'\neq i,k}\max_{j':j'\neq k}\rho_{i'\to}(j')}^{\text{best configuration with no cluster }k},\\ \overbrace{\rho_{k\to k}(k)[k\in neb(c_k')]\cdot\lambda_2\cdot I+[k\in neb(c_k')]\cdot}^{\text{best configuration with a cluster }k}\\ \sum_{\substack{i':s(i',k)\leqslant maxval,\\ i'\neq i,k}}(\rho_{i'\to k}(k)+\lambda_2\cdot I)+\sum_{i'':i\neq i',i,k}\max_{j'}\rho_{i''\to k}(j')\end{array}\right], & \text{for } c_i\neq k\neq i\end{cases} \tag{21}$$

我们可将上述传递的两类消息变量看作一个关于 c_i 的变量和一个常量的和,即

$$\rho_{i\to k}(c_i)=\tilde{\rho}_{i\to k}(c_i)+\bar{\rho}_{i\to k} \tag{22}$$

$$\alpha_{i\leftarrow k}(c_i)=\tilde{\alpha}_{i\leftarrow k}(c_i)+\bar{\alpha}_{i\leftarrow k} \tag{23}$$

于是,上述公式(20)、公式(21)等价于:

$$\rho_{i\to k}(c_i)=s(i,c_i)+s'(i,c_i')+\sum_{k':k'\neq k}\tilde{\alpha}_{i\leftarrow k'}(c_i)+\sum_{k':k'\neq k}\bar{\alpha}_{i\leftarrow k'}(c_i)+[c_i\in neb(c_k')]\cdot\lambda_2\cdot I \tag{24}$$

$$\alpha_{i\to k}(c_i)=\begin{cases}[k\in neb(c_k')]\cdot\sum_{\substack{i':s(i',k)\leqslant maxval,\\ i'\neq k}}(\tilde{\rho}_{i'\to k}(k)+\bar{\rho}_{i'\to k}+\lambda_2\cdot I)+\sum_{i'':i''\neq i',k}\max_{j'}\rho_{i''\to k}(j'), & \text{for } c_i=k=i\\ \sum_{i':i'\neq k}\max_{j':j'\neq k}\tilde{\rho}_{i'\to k}(j')+\sum_{i':i'\neq k}\bar{\rho}_{i'\to k}, & \text{for } c_i\neq k=i\\ \tilde{\rho}_{k\to k}(k)+\bar{\rho}_{k\to k}+[k\in neb(c_k')]\cdot\lambda_2\cdot I+\\ [k\in neb(c_k')]\cdot\sum_{\substack{i':s(i',k)\leqslant maxval,\\ i'\neq i,k}}(\tilde{\rho}_{i'\to k}(k)+\bar{\rho}_{i'\to k}+\lambda_2\cdot I)+\sum_{i'':i\neq i',i,k}\max_{j'}\rho_{i''\to k}(j'), & \text{for } c_i=k\neq i\\ \max\left[\begin{array}{l}\max_{j':j'\neq k}\tilde{\rho}_{k\to k}(j')+\sum_{i':i'\neq i,k}\max_{j':j'\neq k}\tilde{\rho}_{i'\to k}(j')+\sum_{i':i'\neq i}\bar{\rho}_{i'\to k}\tilde{\rho}_{k\to k}(k)+\bar{\rho}_{k\to k}+[k\in neb(c_k')]\cdot\lambda_2\cdot I+\\ [k\in neb(c_k')]\cdot\sum_{\substack{i':s(i',k)\leqslant maxval,\\ i'\neq i,k}}(\tilde{\rho}_{i'\to k}(k)+\bar{\rho}_{i'\to k}+\lambda_2\cdot I)+\sum_{i'':i\neq i',i,k}\max_{j'}\rho_{i''\to k}(j')\end{array}\right], & \\ & \text{for } c_i\neq k\neq i\end{cases} \tag{25}$$

当 $c_i\neq k$ 时, $\rho_{i\to k}(c_i)=\tilde{\rho}_{i\to k}(c_i)+\bar{\rho}_{i\to k}$, 我们假设 $\bar{\rho}_{i\to k}=\max_{j:j\neq k}\rho_{i\to k}(j)$, 则有如下结论:

$$\max_{j':j'\neq k}\tilde{\rho}_{i\to k}(j')=0,\max_{j'}\tilde{\rho}_{i\to k}(j')=\max(0,\tilde{\rho}_{i\to k}(k)) \tag{26}$$

类似地,当 $c_i\neq k$ 时, $\alpha_{i\leftarrow k}(c_i)=\tilde{\alpha}_{i\leftarrow k}(c_i)+\bar{\alpha}_{i\leftarrow k}$, 假设 $\bar{\alpha}_{i\leftarrow k}=\alpha_{i\leftarrow k}$, 同样有:

$$\tilde{\alpha}_{i\leftarrow k}(c_i)=0,\sum_{k':k'\neq k}\tilde{\alpha}_{i\leftarrow k'}(c_i)=\tilde{\alpha}_{i\leftarrow c_i}(c_i) \tag{27}$$

对于 $c_i=k$,公式(27)的和为 0.有了公式(26)和公式(27),我们可进一步对公式(24)和公式(25)进行简化:

$$\rho_{i\to k}(c_i)=\begin{cases} s(i,c_i)+s'(i,c_i')+\sum_{k':k'\neq k}\bar{\alpha}_{i\leftarrow k'}(c_i)++[c_i\in neb(c_k')]\cdot\lambda_2\cdot I, & \text{for } c_i=k \\ s(i,c_i)+s'(i,c_i')+\sum_{k':k'\neq k}\tilde{\alpha}_{i\leftarrow k'}(c_i)+\sum_{k':k'\neq k}\bar{\alpha}_{i\leftarrow k'}(c_i), & \text{for } c_i\neq k \end{cases} \tag{28}$$

$$\alpha_{i\to k}(c_i)=\begin{cases} [k\in neb(c_k')]\cdot\sum_{\substack{i':s(i',k)\leqslant maxval,\\ i'\neq k}}(\tilde{\rho}_{i'\to k}(k)+\bar{\rho}_{i'\to k}+\lambda_2\cdot I)+\sum_{i'':i''\neq i',k}\max(0,\tilde{\rho}_{i''\to k}(j'))+ & \\ \sum_{i'':i''\neq i',k}\bar{\rho}_{i''\to k}, & \text{for } c_i=k=i \\ \sum_{i':i'\neq k}\bar{\rho}_{i'\to k}, & \text{for } c_i\neq k=i \\ \tilde{\rho}_{k\to k}(k)+\bar{\rho}_{k\to k}+[k\in neb(c_k')]\cdot\lambda_2\cdot I+[k\in neb(c_k')]\cdot\sum_{\substack{i':s(i',k)\leqslant maxval,\\ i'\neq i,k}}(\tilde{\rho}_{i'\to k}(k)+\bar{\rho}_{i'\to k}+\lambda_2\cdot I)+ & \\ \sum_{i'':i\neq i',i,k}\max(0,\rho_{i''\to k}(j'))+\sum_{i'':i\neq i',i,k}\bar{\rho}_{i''\to k}, & \text{for } c_i=k\neq i \\ \max\begin{bmatrix} \sum_{i':i'\neq i}\bar{\rho}_{i'\to k} \\ \tilde{\rho}_{k\to k}(k)+\bar{\rho}_{k\to k}+[k\in neb(c_k')]\cdot\lambda_2\cdot I+ \\ [k\in neb(c_k')]\cdot\sum_{\substack{i':s(i',k)\leqslant maxval,\\ i'\neq i,k}}(\tilde{\rho}_{i'\to k}(k)+\bar{\rho}_{i'\to k}+\lambda_2\cdot I)+\sum_{i'':i\neq i',i,k}\max(0,\rho_{i''\to k}(j'))+\sum_{i'':i\neq i',i,k}\bar{\rho}_{i''\to k} \end{bmatrix}, & \\ & \text{for } c_i\neq k\neq i \end{cases} \tag{29}$$

下面求解 $\tilde{\rho}_{i\to k}(c_i=k)=\rho_{i\to k}(c_i=k)-\bar{\rho}_{i\to k}$ 和 $\tilde{\alpha}_{i\leftarrow k}(c_i=k)=\alpha_{i\leftarrow k}(c_i=k)-\bar{\alpha}_{i\leftarrow k}$,以得到更简单的迭代公式:

$$\begin{aligned}\tilde{\rho}_{i\to k}(c_i=k)&=\rho_{i\to k}(c_i=k)-\bar{\rho}_{i\to k}\\ &=\rho_{i\to k}(k)-\max_{j:j\neq k}\rho_{i\to k}(j)\\ &=s(i,c_i)+s'(i,c_i')+\sum_{k':k'\neq k}\bar{\alpha}_{i\leftarrow k'}+[c_i\in neb(c_k')]\cdot\lambda_2\cdot I-\max_{j:j\neq k}\left[s(i,j)+s'(i,c_i')+\tilde{\alpha}_{i\leftarrow j}(j)+\sum_{k':k'\neq k}\bar{\alpha}_{i\leftarrow k'}\right]\end{aligned} \tag{30}$$

$$\begin{aligned}\tilde{\alpha}_{i\leftarrow k}(c_i=k)&=\alpha_{i\leftarrow k}(c_i=k)-\bar{\alpha}_{i\leftarrow k}\\ &=\begin{cases} [k\in neb(c_k')]\cdot\sum_{\substack{i':s(i',k)\leqslant maxval,\\ i'\neq k}}(\tilde{\rho}_{i'\to k}(k)+\bar{\rho}_{i'\to k}+\lambda_2\cdot I)+\sum_{i'':i''\neq i',k}\max(0,\tilde{\rho}_{i''\to k}(j'))+ & \\ \sum_{i'':i''\neq i',k}\bar{\rho}_{i''\to k}-\sum_{j':j'\neq k}\bar{\rho}_{j'\to k}, & \text{for } k=i \\ \tilde{\rho}_{k\to k}(k)+\bar{\rho}_{k\to k}+[k\in neb(c_k')]\cdot\lambda_2\cdot I+[k\in neb(c_k')]\cdot\sum_{\substack{i':s(i',k)\leqslant maxval,\\ i'\neq i,k}}(\tilde{\rho}_{i'\to k}(k)+\bar{\rho}_{i'\to k}+\lambda_2\cdot I)+ & \\ \sum_{i'':i\neq i',i,k}\max(0,\rho_{i''\to k}(j'))+\sum_{i'':i\neq i',i,k}\bar{\rho}_{i''\to k}-\Omega, & \text{for } k\neq i \end{cases}\end{aligned} \tag{31}$$

这里的Ω可以表示为

$$\max\begin{bmatrix} \sum_{j':j'\neq k}\bar{\rho}_{j'\to k} \\ \tilde{\rho}_{k\to k}(k)+\bar{\rho}_{k\to k}+[k\in neb(c_k')]\cdot\lambda_2\cdot I+ \\ [k\in neb(c_k')]\cdot\sum_{\substack{i':s(i',k)\leqslant maxval\\ i'\neq i,k}}(\tilde{\rho}_{i'\to k}(k)+\bar{\rho}_{i'\to k}+\lambda_2\cdot I)+\sum_{i'':i\neq i',i,k}\max(0,\rho_{i''\to k}(j'))+\sum_{i'':i\neq i',i,k}\bar{\rho}_{i''\to k} \end{bmatrix}.$$

从而,等式(31)可进一步化简为

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$$\begin{cases}[k\in neb(c_k')]\cdot\sum\limits_{\substack{i':s(i',k)\leqslant maxval,\\ i'\neq k}}(\tilde{\rho}_{i'\to k}(k)+\lambda_2\cdot I)+\sum\limits_{i'':i''\neq i',k}\max(0,\tilde{\rho}_{i''\to k}(j')), & \text{for } k=i\\ \tilde{\rho}_{k\to k}(k)+[k\in neb(c_k')]\cdot\lambda_2\cdot I+[k\in neb(c_k')]\cdot\sum\limits_{\substack{i':s(i',k)\leqslant maxval,\\ i'\neq i,k}}(\tilde{\rho}_{i'\to k}(k)+\lambda_2\cdot I)+\sum\limits_{i'':i\neq i',i,k}\max(0,\rho_{i''\to k}(j'))- & \\ \max\left[0,\tilde{\rho}_{k\to k}(k)+[k\in neb(c_k')]\cdot\lambda_2\cdot I+[k\in neb(c_k')]\cdot\sum\limits_{\substack{i':s(i',k)\leqslant maxval,\\ i'\neq i,k}}(\tilde{\rho}_{i'\to k}(k)+\lambda_2\cdot I)+\sum\limits_{i'':i\neq i',i,k}\max(0,\rho_{i''\to k}(j'))\right], & \text{for } k\neq i\end{cases} \tag{32}$$

在更新过程中,当 $c_i\neq k$ 时,$\tilde{\rho}_{k\to i}(c_i)$ 和 $\tilde{\alpha}_{i\leftarrow k}(c_i)$ 并没有使用到(且注意到 $\tilde{\alpha}_{i\leftarrow k}(c_i\neq k)=0$),因此,传递的消息可以认为是 $r(i,k)=\tilde{\rho}_{k\to i}(k)$ 和 $a(i,k)=\tilde{\alpha}_{i\leftarrow k}(k)$:

$$r(i,k)=s(i,k)+s'(i,k)+[k\in neb(c_k')]\cdot\lambda_2\cdot I-\max_{j:j\neq k}[s(i,j)+s'(i,j)+a(i,j)] \tag{33}$$

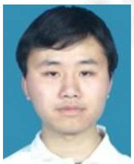
$$a(i,k)=\begin{cases}[k\in neb(c_k')]\cdot\sum\limits_{\substack{i':s(i',k)\leqslant maxval\\ i'\neq k}}(r(i',k)+\lambda_2\cdot I)+\sum\limits_{i'':i''\neq i',k}\max(0,r(i'',k)), & k=i\\ \min\left[0,\ [k\in neb(c_k')]\cdot\sum\limits_{\substack{i':s(i',k)\leqslant maxval\\ i'\neq i}}(r(i',k)+\lambda_2\cdot I)+\sum\limits_{i'':i''\neq i'}\max(0,r(i'',k))\right], & k\neq i\end{cases} \tag{34}$$

公式(34)就是文中给出的迭代公式.在迭代计算 $a(i,k)$更新时,min[0,·]的计算由如下等式得到:

$$x-\max(0,x)=\min(0,x).$$

杭文龙(1988－),男,江苏南通人,博士生,CCF 学生会员,主要研究领域为模式识别,数据挖掘.

刘解放(1982－),男,博士生,主要研究领域为模式识别,数据挖掘.



蒋亦樟(1988－),男,博士,讲师,CCF 专业会员,主要研究领域为模式识别,系统建模.

王士同(1964－),男,教授,博士生导师,主要研究领域为模式识别,人工智能.

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