分层点云的分段化*

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Segmentation from Stratified Range Image

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Abstract: This paper addresses an approach of segmenting dense range. It is the most difficult and important step of constructing a solid model from dense range obtained from 3D scanner. The original object is assumed to be covered by piecewise quadric surfaces and planes, and the range image can be stratified in the way that the points on different layers are situated on parallel planes respectively. The method is based on the strategy "from one dimension manifold to two dimension manifold". Quadric curve segments and straight line segments on every layer is recognized firstly. Group the quadric curve and straight line segments situated on a sequence of adjacent parallel planes with some coherent properties. The quadric surfaces or planes bounding the original object are composed by the quadric curve and straight line segments in same groups. Three examples show the algorithm's performance. This approach can be applied in reverse engineering to construct solid model of mechanical parts.

Key words: range image; segmentation; solid modeling; geometric model; quadric surface

摘 要: 说明一种点云的分段算法.这是从点云重建立体造型的关键步骤.假设原对象是以平面和二次曲面为边界,且点云是分布在一些平行平面的层上.提出的算法是基于从一维流形发展到二维流形的策略.首先在每一层上识别二次曲线和直线,把位于相邻层且有相同固有属性的二次曲线或直线分成组,每一组内的二次曲线或直线位于同一个二次曲面或平面上.给出了 3 个例子说明该算法的效果.该方法可用于逆向工程构造产品的立体造型.

关键词: 点云;分段;立体造型;几何模型;二次曲面

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To reconstruct solid modeling from digitized 3D points appears in many areas, particularly in engineering design, manufacturing and analysis. The existing CAD/CAM/CAE system is operated on an object represented by geometric model (such as solid model). An object represented by dense range image is impossible to be operated by CAD/CAM/CAE systems because range image contains only points cloud, no structure at all. The only approach to apply those systems is to reconstruct a representation of geometric model for the object from range image. The key step of the reconstruction is to recognize the boundary surfaces of the object.

Range image segmentation is to logically divide the original point set into subsets, one for each natural surface (free surface, quadric surface and etc.) so that each subset contains just those points sampled from a particular natural surface. Támás Várady surveyed the strategies for range image segmentation and outlined the most important algorithmic steps[1]. The published algorithms can be divided into three categories[2]. They are Split-and-merge[3], algorithm based on clustering[4] and Region growing[5,6]. All of these approaches are based on manipulating on two-dimension manifold. In this paper we assume that the 3D points in the range image can be layered. The points belonging to a layer are situated on one plane. The planes are parallel, and the distances between two adjacent planes are equal. This assumption is natural for some 3D scanners. The 3D points on different layers represent the intersection curves between the object and a sequence of parallel planes.

We also assume that the object is bounded by piecewise planes and quadric surfaces. Since the intersection between a quadric surface and a plane is a quadric curve, the points on one layer can be approximated by line segments and quadric curve segments.

In this paper the above assumptions are assumed to be true. The algorithm presented is based on the strategy "from one dimension manifold to two-dimension manifold" rather than manipulating on two-dimension manifold directly. After segmenting each layer we obtain line and quadric curve segments in a sequence of parallel planes. According to the coherent properties of the sequence of quadric curves we can determine whether planes or quadric surfaces can approximate some adjacent line and quadric curve segments.

It is well known that to manipulate on one-dimension manifold is much more simple and accurate than to do the same job on two-dimension manifold. From analytic geometry to determine a quadric surface to approximate a sequence of quadric curves with same coherent properties is a simple matter. The above idea and its implementation are the original contribution and the motivation of this research.

1 Segmentation of Points on One Layer

In the previous paragraph the range image is grouped into layers. Since the 3D points on one layer represent intersection of the boundary of the object and a plane, they can be approximated by straight line and quadric curve segments. Segmentation of one layer means to segment the points into subsets that each subset represents an

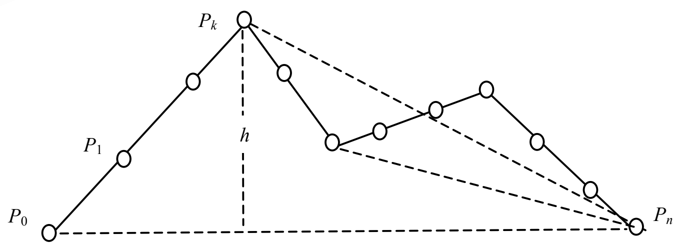

Fig.1 Recogniting straight-line segments between points P_0 and P_n

individual straight line or quadric curve segment. In order to withdraw the straight-line segments between points P_0 and P_n, link two end points P_0P_n. If the distances of all the points $P_1,P_2,\ldots,P_{n-1}$ to line segment P_0P_n are less than a given tolerance, the points P_0 to P_n are situated on one straight line segment, otherwise find the farthest point P_k from line P_0P_n, and do the same procedures for P_0P_k and P_kP_n. In this way a polygon with block lines in Fig.1 is created. The long sides are the straight line segments. Usually the short sides represent some quadric curves.

A Least-squares method is applied to find a quadric curve approximating a sequence of points (x_i,y_i), i=0,1,…,m. Generally, the equation of a quadric curve is

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0\ . \tag{1}$$

In order to find the coefficients of (1) let

$$I = \sum_{i=0}^{m} (Ax_i^{\ 2} + Bx_iy_i + Cy_i^{\ 2} + Dx_i + Ey_i + F)^2\ . \tag{2}$$

A, B, C, D, E and F are the solution of the following simultaneous liner equations

$$\frac{\partial I}{\partial A}=0,\ \frac{\partial I}{\partial B}=0,\ \frac{\partial I}{\partial C}=0,\ \frac{\partial I}{\partial D}=0,\ \frac{\partial I}{\partial E}=0,\ \frac{\partial I}{\partial F}=0\ . \tag{3}$$

But (3) is a homogeneous simultaneous equations system. It has only zero solution. Since the curve (1) is invariable, if all the coefficients of (1) are modified by a constant fact, to overcome this problem instead of solving (3) we can solve a non-linear programming as

$$\text{Min}\quad I \tag{4}$$

with constraint

$$A^2 + B^2 + C^2 + D^2 + E^2 + F^2 = 1\ . \tag{5}$$

Let $U = (A,B,C,D,E,F)$ and

$$I = UMU^T\ , \tag{6}$$

where M is a 6×6 symmetric matrix, and every eigenvalue of M is positive. The eigenvector of the smallest eigenvalue λ of matrix M is the solution of (4) and (5)[11], and Min $I=\lambda$. The eigenvector can be computed by several methods from numerical analysis textbook.

If a sequence of points represent several linked quadric curve segments, we cannot simply use (4) and (5) to find the individual quadric curve segment. The sequence of points must be segmented into several subsets, and each subset can be approximated by an individual quadric curve. To realize the segmentation once we find that a sequence of points can not be approximated by a quadric curve properly, the points are sub-divided into two sets by the middle point. If none of them can be approximated by a quadric curve properly, the subsets are divided further. If one of the subset can be approximated properly, extend the subset by adding adjacent points to the subset, and test the extended set again, until the subset becomes the largest set of points being approximated by a quadric curve properly. Experimental results show that by applying this method a sequence of points even representing two C^1 continue quadric curves can be segmented accurately.

2 Segmentation of Range Image

Now we are in a position to describe the approach of identifying a quadric surface from the quadric curves of some adjacent layers. The intersection of a quadric surface and a plane is a quadric curve. Quadric curves can be divided into three types: ellipse, parabolic and hyperbolic. All the intersections of a quadric surface and a sequence of parallel planes belong to a same type of quadric curves. There are nine different quadric surfaces altogether. They are ellipsoid, ellipse cylinder, hyperbolic cylinder, parabolic cylinder, cone, hyperboloid of one sheet, hyperboloid of two sheets, elliptic paraboloid and hyperbolic paraboloid. From the previous paragraph we know that the points on one layer can be approximated by quadric curve segments and line segments. From some adjacent layers we can find a sequence of quadric curves which have same coherent properties. Analyzing the intersection between quadric surfaces and parallel planes, altogether twenty-two distinguishable cases can be found. According to the type of intersection quadric curves they are classified to four groups.

2.1 The ntersections are straight lines

If a group of parallel straight line segments can be segmented from the intersection curves situated on a sequence of parallel planes, there are four probabilities.

• If the intersection points of the group of straight lines and a perpendicular plane are situated on a straight line, a plane can approximate the group of straight lines. See Fig.2.

• If the intersection points of the group of straight lines and a perpendicular plane are situated on an ellipse, the lines can be approximated by an ellipse cylinder. See Fig.3.

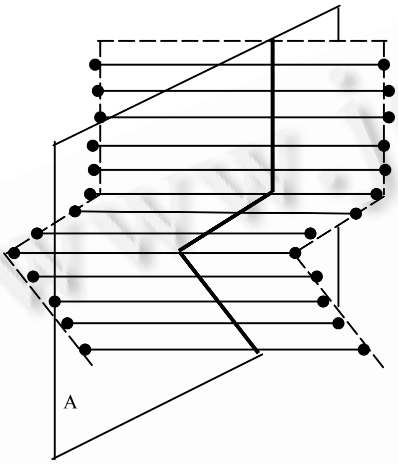

Fig.2 The intersections of a sequence of parallel lines and a perpendicular plane A are on a straight line. It illustrates that there must be a plane here.

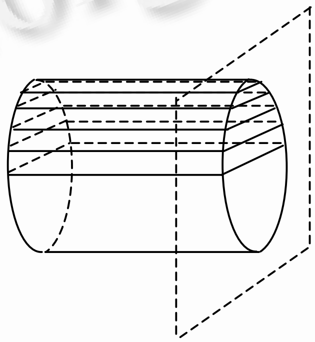

Fig.3 The intersection points of the parallel lines and one of their perpendicular plane are actually situated on a ellipse.

• If the intersection points of the group of straight lines and a perpendicular plane are situated on a hyperbolic, a hyperbolic cylinder can approximate the lines.

• If the intersection points of the group of straight lines and a perpendicular plane are situated on a parabolic, a parabolic cylinder can approximate the lines.

2.2 The intersections are ellipse curves

If a group of ellipse segments is segmented from the intersection curves situated on a sequence of parallel planes, and if they have the following coherent properties, the ellipse curves can be approximated by a quadric surface. The eccentricity e of the ellipse curves is constant. The major axes of the ellipse curves have the same direction *D*. The centers of the ellipse curves are on a straight line. There are six probabilities.

• If the vertices of the long axis of the ellipse curves are situated on two straight lines, and the two lines intersect, a cone can approximate the ellipse curves.

• If the vertices of the long axis of the ellipse curves are situated on two straight lines, and the two lines are parallel, the ellipse curves can be approximated by an ellipse cylinder.

• If the vertices of the long axis of the ellipse curves are situated on an ellipse, the ellipse curves can be approximated by an ellipsoid. See Fig.4.

• If the vertices of the long axis of the ellipse curves are situated on a parabolic, the ellipse curves can be approximated by an elliptic paraboloid.

• If the vertices of the long axis of the ellipse curves are situated on a same sheet of a hyperbolic, the ellipse curves can be approximated by a hyperboloid of two sheets.

• If the two vertices of the long axis of the ellipse curves are situated on two sheets of a hyperbolic respectively, the ellipse curves can be approximated by a hyperboloid of one sheet.

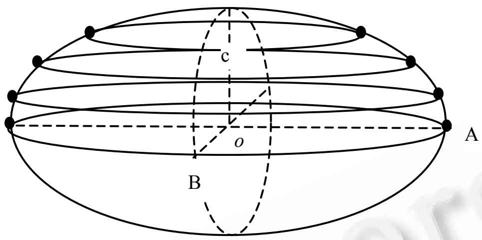

Fig.4 The ellipse curves can be approximated by an ellipsoid if their vertices are situated on an ellipse

2.3 The intersections are parabolic curves

If a group of parabolic curves is segmented from the intersection curves situated on a sequence of parallel planes, and if they have the following coherent properties, the parabolic curves can be approximated by a quadric surface. The parameters p (the distance from focus to directrix) of the parabolic curves are constant or liner as the plane situated changes. The axes of the parabolic curves have a same direction D.

There are six probabilities.

• If the vertices of the parabolic curves are situated on a straight line, and the parameter p is linear, the parabolic curves can be approximated by a cone.

• If the vertices of the parabolic curves are situated on a straight line, and the parameter p is constant, the parabolic curves can be approximated by a parabolic cylinder.

• If the vertices of the parabolic curves are situated on an another parabolic, the inner product of the axis direction of the parabolic and D is greater than zero, and the parameters p are constant, the parabolic curves can be approximated by an elliptic paraboloid.

• If the vertices of the parabolic curves are situated on an another parabolic, the inner product of the axis direction of the parabolic and D is less than zero, and the parameters p are linear, the parabolic curves can be approximated by a hyperbolic paraboloid.

• If the vertices of the parabolic curves are situated on a hyperbolic, the inner product of the axis direction of the hyperbolic and D is less than zero, and the parameters p are linear, the parabolic curves can be approximated by a hyperboloid of one sheet.

• If the vertices of the parabolic curves are situated on a hyperbolic, the inner product of the axis direction of the hyperbolic and D is greater than zero, and the parameters p are linear, the parabolic curves can be approximated by a hyperboloid of two sheets.

The axis direction of one sheet of hyperbolic is supposed to be the direction along the transverse axis from the vertex and pointing to the extending direction of the sheet of the hyperbolic.

2.4 The intersections are hyperbolic curves

If a group of hyperbolic curves is segmented from the intersection curves situated on a sequence of parallel planes, and if they have the following coherent properties, the hyperbolic curves can be approximated by a quadric surface. The eccentricity e of the hyperbolic curves is constant. The transverse axes of the hyperbolic curves have the same direction D. The centers of the hyperbolic curves are on a straight line. There are five probabilities.

• If the two vertices of the hyperbolic curves are situated on two straight lines, and the two lines are intersected, the hyperbolic curves can be approximated by a cone.

• If the two vertices of the hyperbolic curves are situated on two straight lines, and the two lines are parallel, the hyperbolic curves can be approximated by a hyperbolic cylinder.

• If the two vertices of the hyperbolic curves are situated on a parabolic curve, the hyperbolic curves can be approximated by a hyperbolic paraboloid.

• If the two vertices of the hyperbolic curves are situated on two sheets of another hyperbolic curve respectively, the hyperbolic curves can be approximated by a hyperboloid of two sheets.

• If the two vertices of the hyperbolic curves are situated on one sheet of an another hyperbolic, the hyperbolic curves can be approximated by a hyperboloid of one sheet.

• If the two vertices of the hyperbolic curves are situated on an ellipse, the hyperbolic curves can be approximated by a hyperboloid of one sheet.

Using the knowledge of analysis geometry we can prove the above illations. In most of the case, only cylinders, cones and spheres are considered since they are more important in mechanical engineering. For most of the cases the quadric surface approximating a sequence of quadric curves can be computed from the sequence of quadric curves immediately, it is efficient and precise by this way. The quadric surface can also be computed by the Least-squares method. The equation of a quadric surface is

$$Ax^2 + By^2 + Cz^2 + Dxy + Exz + Fyz + Gx + Hy + Iz + J = 0 . \tag{7}$$

From the above analysis, the 3D points on one quadric surface can be segmented from the range image. Assume that the 3D points are (x_i, y_i, z_i) i=0,1,2,…,m. Let

$$K = \sum_{i=0}^{m}(Ax_i^2 + By_i^2 + Cz_i^2 + Dx_iy_i + Ex_iy_i + Fy_iz_i + Gx_i + Hy_i + Iz_i + J)^2 .$$

The coefficients A, B, C, D,…,J can be obtained by solving the following programming.

$$\text{Min} \quad K \tag{8}$$

with constraint

$$A^2 + B^2 + C^2 + D^2 + E^2 + F^2 + G^2 + H^2 + I^2 + J^2 = 1 . \tag{9}$$

3 Reconstruction of a Solid Model

Once the boundary surfaces are found, the intersection lines between quadric surfaces are calculated. The approximate edges of objects can be also obtained after using the segmentation described above. So long as the edges are obtained, the Wing-edge structure can be applied to create a solid model for the object.

4 Examples

Example A. Original data of the object contain 256×93 points. Figure 5 illustrates the layered data and Fig.6 shows the B-rep model of the object. Computing time is 55 seconds on Pentium III 550MHZ.

Example B. Original data of the object contain 256×93 points. Figure 7 illustrates the layered data and Fig.8 shows the B-rep model of the object. Computing time is 55 seconds on Pentium III 550MHZ.

Example C. Original data of the object contain 512×60 points. Figure 9 illustrates the layered data and Fig. 10 shows the B-rep model of the object. Computing time is 70 seconds to complete the process on a computer with CPU Pentium III 550MHZ and 128M memory.

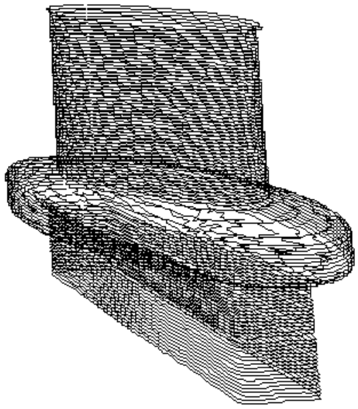

Fig.5 The original data of example A was grouped into 93 layers

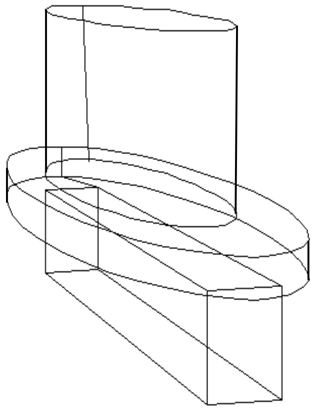

Fig.6 The identified B-rep model of example A

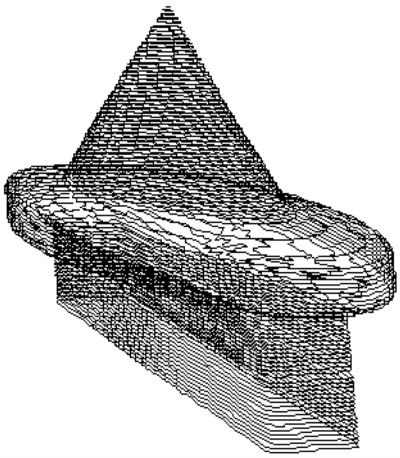

Fig.7 The original data of example B was grouped into 93 layers

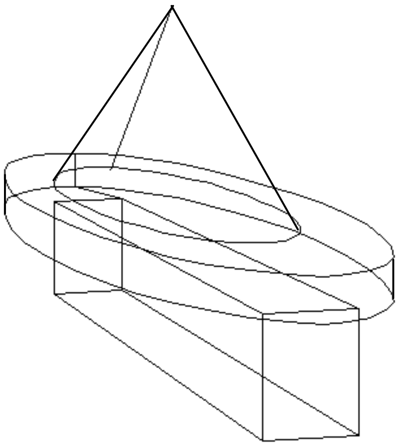

Fig.8 The identified B-rep model of example B

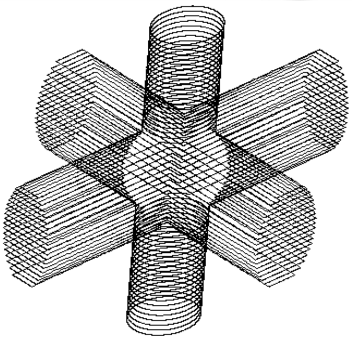

Fig.9 The original picture of example C

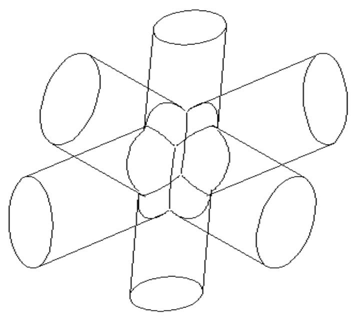

Fig.10 The identified B-rep model of example C

5 Reconstruction of a Solid Model

A new segmentation method based on stratified range image has been described in this paper. It is efficient for constructing a solid model from range image. “from one dimension manifold to two dimension manifold” is the main strategy. It can be more accurate and easier. How to segment a free surface by the strategy is an open question

Generally speaking, the matrix of the equations generated by least square can be ‘bad’ matrix for some critical data. That is the condition number is very small. For example if the points are situated on a 5 degree circle arc, using least square to find the center of the arc, the error can be as large as 0.03. If the arc is greater than 10 degree, the error is less than 0.0002. Although the small degree arc does not appear often, and this small part usually can be considered as part of the adjacent surface, it is helpful to understand the effect raised by the ‘bad’ matrix.

References:

[1] Varady T, Martin RR, Cox J. Reverse engineering of geometric models: an introduction. Computer-Aided Design, 1997,29(4): 255~268.

[2] Fitzgibbon AW, Eggert D, Fisher RB. High-Level CAD model acquisition from range image. Computer-Aided Design, 1997,29(4): 321~330.

[3] Faugeras O, Hebert M. A 3-d recognition and positioning algorithm using geometric matching between primitive surfaces. International Journal of Robotics Research, 1986,5(3):27~52.

[4] Flynn P. CAD-Based computer vision: Modeling and recognition strategies [Ph.D. Thesis]. Michigan State University, 1990.

[5] Besl PJ, Tain RC. Segmentation through variable-order surface fitting. IEEE PAMI, 1988,10(2):167~192.

[6] Spaidis NS, Besl PJ. Direct construction of polynomial surfaces from dense range images through region growing. ACM Transactions on Graphics, 1995,14(2):171~200.

[7] Chivate PN, Jablokow AG. Solid-Modeling generation from measured point data. Computer-Aided Design, 1993,25(9):587~600.

[8] Hoffman CM. Geometric and Solid Modeling: An Introduction. San Mateo, CA: Morgan Kaufman, 1989.

[9] Hoover A, Goldgof D, Bowyer, KW. Extracting a valid boundry representation from a segmented range images. IEEE PAMI, 1995, 17(9):920~924.

[10] Milroy MJ, Bradley Vickers GW. Segmentation of a wrap-around model using an active contour. Computer-Aided Design, 1997, 29(4):299~320.

[11] Richard IH. In defense of 8-point algorithm. IEEE PAMI, 1997,19(6):580~593.