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Shape Modification of NURBS Surfaces via Constrained Optimization*

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Abstract: A new method for shape modification of NURBS surfaces is proposed in this paper. Explicit formulae for computing new control points are derived by using constrained opimization method. Examples are also given to compare the results of Piegl's method with those of the new method.

Key words: NURBS surfaces; shape modification; constrained optimization

NURBS curves and surfaces are widely used in curve and surface design[1,2], and it's always an interesting problem to edit and modify their shape[3~6]. By definition of NURBS curves and surfaces, there are three ways to modify the shape:

- change knot vectors;
- move control points;
- change the weights.

The objective of this paper is to present a new method for modifying the shape of NURBS surfaces more naturally by moving control points.

Piegl has proposed a way to modify the shape of NURBS curves[3] and surfaces[4] called "control point-based modification". For a given NURBS curve or surface and a given target point, how to modify the control points such that the original curve or surface can pass through the target point? Piegl gave an efficient way by moving one control point and presented an explicit formula to compute the new control point. But due to just one control point modification, the shape modification seems to be unnatural when the target point is not near the curve or surface. Therefore a new method for shape modification of NURBS curve has been proposed in Ref. [9], where more than one control point is modified, so the modification seems to be natural.

In this paper, we present a solution for local shape modification of surface by using constrained optimization

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method which allows more than one control point to be changed. The new method not only makes the surface pass through the target point, but also makes it possible to minimize shape change in certain sense. We also derive an explicit fomula to compute new control points. In addition, we will consider the multi-target problem of shape modification.

1 Local Shape Modification

1.1 Problem statement

A NURBS surface with control points P_{ij}, $0\leqslant i\leqslant n, 0\leqslant j\leqslant m$, can be defined as

$$p(u,v)=\frac{\sum_{i=0}^{n}\sum_{j=0}^{m}\omega_{ij}P_{ij}N_{i,k}(u)N_{j,l}(v)}{\sum_{i=0}^{n}\sum_{j=0}^{m}\omega_{ij}N_{i,k}(u)N_{j,l}(v)} \tag{1}$$

$$u_{k-1}\leqslant u\leqslant u_{n+1}, v_{l-1}\leqslant v\leqslant v_{m+1},$$

where ω_{ij} are corresponding weights of P_{ij}, $N_{i,k}(u)$ and $N_{j,l}(v)$ are the normalized B-spline base functions of orders k and l, respectively, defined over knot vectors

$$U=\{u_0,u_1,\ldots u_k,\ldots u_n,u_{n+1},\ldots u_{n+k}\}$$

and

$$V=\{v_0,v_1,\ldots,v_l,\ldots v_m,v_{m+1},\ldots v_{m+l}\}.$$

Usually, we set $u_0=u_1=\ldots=u_{k-1}=0$, $u_{n+1}=u_{n+2}=\ldots=u_{n+k}=1$, $v_0=v_1=\ldots=v_{l-1}=0$, $v_{m+1}=v_{m+2}=\ldots=v_{m+l}=1$. By using symbols of rational based function, Eq. (1) can be rewritten as

$$p(u,v)=\sum_{i=0}^{n}\sum_{j=0}^{m}P_{ij}R_{i,j}(u,v)\quad 0\leqslant u,v\leqslant 1, \tag{2}$$

where

$$R_{i,j}(u,v)=\frac{\omega_{ij}N_{i,k}(u)N_{j,l}(v)}{\sum_{i=0}^{n}\sum_{j=0}^{m}\omega_{ij}N_{i,k}(u)N_{j,l}(v)}$$

For a start point S in surface $P(u,v)$, (u_s,v_s) is the papameter of S, and T is the target point. We hope to modify the surface such that it passes through point T. In geometric modeling systems, we usually pick up a point in surface, then drag the surface to a target point by mouse, so (u_s,v_s) is unknown. But we can compute it by using the algorithm presented in Ref. [10].

For Piegl's method, the control point which has more influence should be determined firstly by using relation between (u_s,v_s) and knot vectors U,V. Then this point is adjusted to satisfy shape requirement, i.e., $T=\hat{P}(u_s,v_s)$, where $\hat{P}(u,v)$ is the modified surface. However, for a NURBS surface of order $k\times l$, just $k\times l$ patches will be modified. When distance between S and T, denoted by $D(S,T)$, is large, the new surface will not satisfy the fairness requirement.

1.2 Constrained optimization solution

A reasonable solution is to determine how many control points should be adjusted by relation between $D(S,T)$ and the shape of control net. We give explicit formulae for local shape modification by adjusting more than one control point.

Suppose control points $P_{ij}, i_1\leqslant i\leqslant i_2, j_1\leqslant j\leqslant j_2$, are to be changed. We choose perturbation $\varepsilon_{ij}=[\varepsilon_{ij}^x,\varepsilon_{ij}^y,\varepsilon_{ij}^z]^T$ $(i_1\leqslant i\leqslant i_2,\ j_1\leqslant j\leqslant j_2)$ for those control points, such that the modified surface

$$\hat{P}(u,v)=\sum_{i=0}^{n}\sum_{j=0}^{m}\hat{P}_{ij}R_{i,j}(u,v)=\sum_{i=0}^{n}\sum_{j=0}^{m}P_{ij}R_{i,j}(u,v)+\sum_{i=i_1}^{i_2}\sum_{j=j_1}^{j_2}\varepsilon_{ij}R_{i,j}(u,v)\quad 0\leqslant u,v\leqslant 1 \tag{3}$$

passes through the target point T, i.e., satisfies the following equation

$$T=\hat{P}(u_s,v_s)=\sum_{i=0}^{n}\sum_{j=0}^{m}P_{ij}R_{i,j}(u_s,v_s)+\sum_{i=i_1}^{i_2}\sum_{j=j_1}^{j_2}\varepsilon_{ij}R_{i,j}(u_s,v_s)=S+\sum_{i=i_1}^{i_2}\sum_{j=j_1}^{j_2}\varepsilon_{ij}R_{i,j}(u_s,v_s)$$

where $\hat{P}(u,v)$ is the objective surface.

We determine ε_{ij}, $(i_1\leqslant i\leqslant i_2,\ j_1\leqslant j\leqslant j_2)$ by the constrained optimization method. The optimization objective is

$$\sum_{i=i_1}^{i_2}\sum_{j=j_1}^{j_2}\|\varepsilon_{ij}\|^2=\text{Min}, \tag{4}$$

and the Lagrange function is defined by

$$L=\sum_{i=i_1}^{i_2}\sum_{j=j_1}^{j_2}\|\varepsilon_{ij}\|^2+\lambda(T-\hat{P}(u_s,v_s)) \tag{5}$$

where $\lambda=[\lambda_1,\lambda_2,\lambda_3]$ is the Langrange multiplier and $\|\cdot\|$ is Euclidean norm.

Let $\frac{\partial}{\partial\lambda_1}(L)=\frac{\partial}{\partial\lambda_2}(L)=\frac{\partial}{\partial\lambda_3}(L)=0$, $\frac{\partial}{\partial\varepsilon_{ij}^x}(L)=\frac{\partial}{\partial\varepsilon_{ij}^y}(L)=\frac{\partial}{\partial\varepsilon_{ij}^z}(L)=0$ for $i_1\leqslant i\leqslant i_2$, $j_1\leqslant j\leqslant j_2$, and write the derived formula in vector form, then we have the following system

$$\begin{cases}T=S+\sum_{i=i_1}^{i_2}\sum_{j=j_1}^{j_2}\varepsilon_{ij}R_{i,j}(u_s,v_s)\\ \varepsilon_{ij}=\frac{\lambda}{2}R_{i,j}(u_s,v_s), \quad i_1\leqslant i\leqslant i_2,\ j_1\leqslant j\leqslant j_2\end{cases} \tag{6}$$

By solving the above equation system, we finally get the explicit solution as follows.

$$\varepsilon_{ij}=\frac{R_{i,j}(u_s,v_s)}{\sum_{i=i_1}^{i_2}\sum_{j=j_1}^{j_2}R_{i,j}^2(u_s,v_s)}(T-S), \tag{7}$$

and the objective curve $\hat{P}(u,v)$ can be obtained by substituting Eq. (7) into Eq. (3).

1.3 Comparison and examples

If only one control point is modified, from Eq. (7), we have

$$\varepsilon_{ij}=\frac{T-S}{R_{i,j}(u_s,v_s)}$$

It's just the Eq. (30) in Piegl's paper[4].

As mentioned in the above section, when $D(S,T)$ is large enough, the number of modified control points should be determined according to relation between $D(S,T)$ and the shape of control polygon. There are two solutions:

(1) by relation between $D(S,T)$ and the sum of lengths of line segments in control net;

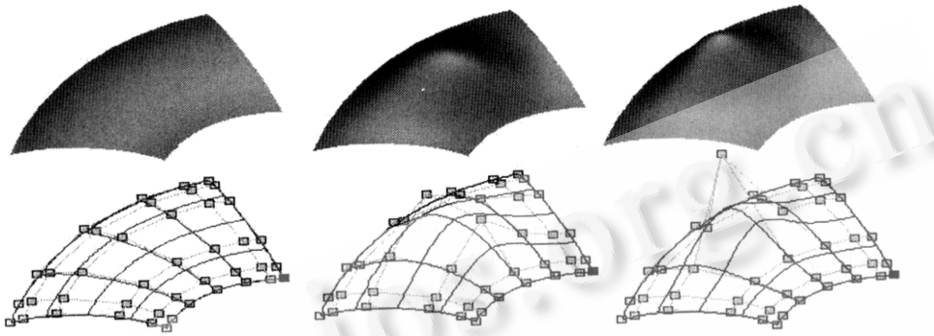
(2) based on relation between target point T and convex hull of the original surface. If the target T doesn't lie in the convex hull of surface, all control points should be modified.

If all control points are modified, by setting $i_1=0$, $i_2=n+1$, $j_1=0$, $j_2=m+1$ in Eq. (7), we get the solution

$$\varepsilon_{ij}=\frac{R_{i,j}(u_s,v_s)}{\sum_{i=0}^{n}\sum_{j=0}^{m}R_{i,j}^2(u_s,v_s)}(T-S) \tag{8}$$

However, by local support properties of B-spline basis, it's not necessary to adjust all control points. Suppose $[u_q,u_{q+1}]\times[v_r,v_{r+1}]$ is the rectangle which contains (u_s,v_s), and u_q,v_r are knots. Actually, for parameter (u_s,v_s), only R_{ij}, $(q-k-1\leqslant i\leqslant q, r-l+1\leqslant j\leqslant r)$, are not zero. So a reasonable solution is to adjust $k\times l$ control points, i.e., P_{ij}, $q-k+1\leqslant i\leqslant q$, $r-l+1\leqslant j\leqslant r$.

The following Fig. 1 is an example which demonstrates the effect of shape modification by the new method, where (a) is shading and wireframe representation of the original surface, (b) is those of modified surface by the new method, and (c) is those of modified surface by Piegl's method.



(a) (b) (c)

Fig. 1 Comparison of two methods

2 Multi-Target Problem

For $k\times l$ order NURBS surface $P(u,v)$ and target points $T_l, l=0,1,\dots r$, how to adjust control points such that the modified surface $P(u,v)$ passes through those target points?

By projecting point T_l to surface $P(u,v)$, the corresponding parameter (u_l,v_l) can be obtained. Then we choose perturbation $\varepsilon_{ij}=[\varepsilon_{ij}^x,\varepsilon_{ij}^y,\varepsilon_{ij}^z]^T$ for every control point P_{ij}, such that the modified surface

$$\hat{P}(u,v)=\sum_{i=0}^{n}\sum_{j=0}^{m}(P_{ij}+\varepsilon_{ij})R_{i,j}(u,v),\quad 0\leqslant u,v\leqslant 1 \tag{9}$$

satisfies the shape requirement

$$T_l=\hat{P}(u_l,v_l)=\sum_{i=0}^{n}\sum_{j=0}^{m}(P_{ij}+\varepsilon_{ij})R_{i,j}(u_l,v_l),\quad l=0,1,\dots,r. \tag{10}$$

From optimization objective

$$\sum_{i=0}^{n}\sum_{j=0}^{m}\|\varepsilon_{ij}\|^2=\mathrm{Min}, \tag{11}$$

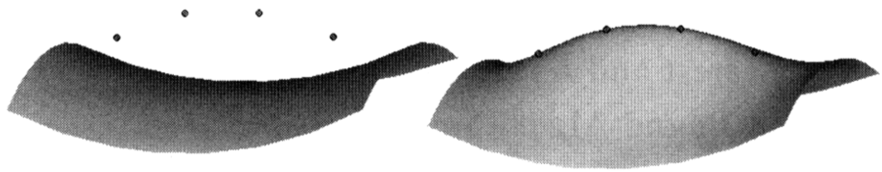
and Lagrange function

$$L=\sum_{i=0}^{n}\sum_{j=0}^{m}\|\varepsilon_{ij}\|^2+\sum_{l=0}^{r}\lambda_l(T_l-\hat{P}(u_l,v_l)), \tag{12}$$

the following equation system can be obtained

$$\begin{cases} T_l=\sum_{i=0}^{n}\sum_{j=0}^{m}(P_{ij}+\varepsilon_{ij})R_{i,j}(u_l,v_l), & l=0,q,\dots,r \\ 2\varepsilon_{ij}+\sum_{l=0}^{r}\lambda_l R_{i,j}(u_l,v_l)=0, & i=0,1,\dots,n \end{cases} \tag{13}$$

From the above equation system, the constrained optimization solution can be obtained. The following Fig. 2 is an example which shows the effect of shape modification for multi-target points, where (a) is shading representation of the original surface, (b) is that of modified surface.



(a) (b)

Fig. 2 Modification with multi-target

3 Conclusion

In this paper, we propose a new method for shape modification of NURBS surfaces such that the modified surface passes through a given target point more naturally. By using constrained opimization method, more than one control point is allowed to be changed.

Explicit formulae are derived to compute new control points. In addition, we have also discussed shape modification problem with multi-target points, in which constrained optimization solution can be obtained by solving an equation system.

References:

[1] Piegl, L., Tiller, W. The NURBS Book. Springer-Verlag, Berlin, Heidelberg, New York, 1995.
[2] Farin, G. NURBS curves and surface from projective geometry to practical use. In: Peters, A. K., ed. Wellesley, MA, 1995.
[3] Piegl, L. Modifying of the shape of rational B-spline, part 1: curves. Computer Aided Design, 1989,21(8):509~518.
[4] Piegl, L. Modifying of the shape of rational B-spline, part 2: surfaces. Computer Aided Design, 1989,21(9):538~546.
[5] Terzopoulos, D., Qin, H. Dynamic NURBS with geometric constraints for interactive sculpting. ACM Transactions on Graphics, 1994,13(2):103~106.
[6] Zheng, J. M., Chan, K. W., Gibson, I. A new approach for direct manipulation of free-form curve. Computer Graphics Forum, 1998,17(3):327~334.
[7] Hsu, W. M., Hughes, J. F., Kaufman, H. Direct manipulation of free-form deformation. Computer Graphics, 1992, 26(4):177~183.
[8] Celniker, G., Gossard, D. Deformable curve and surface finite-elements for free-form shape design. Computer Graphics, 1991,25(4):327~334.
[9] Hu Shi-min, Zhou Deng-wen, Sun Jia-guang. Shape modification of NURBS curves via constrained optimization. In: Chen Hou-peng ed. Proceedings of CAD/Graphics'99. Shanghai: Wen Hui Publishers, 1999. 958~962.
[10] Hu Shi-min, Sun Jia-guang, Jin Tong-guang, *et al*. Computing the parameters of points in NURBS curves and surfaces via moving affine frame method. Journal of Software, 2000,11(1):49~53 (in Chinese).

附中文参考文献:

[10] 胡事民,孙家广,金通光,等. 基于活动仿射标架反求 Nurbs 曲线/曲面参数. 软件学报,2000,11(1):49~53.

基于约束优化的 NURBS 曲面形状修改

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摘要: 提出了一种修改 NURBS 曲面形状的新方法. 利用约束优化方法,得到计算曲面新的控制顶点的显式公式,并给出例子以比较所提出的方法和 Piegl 方法的效果.

关键词: NURBS 曲面;形状修改;约束优化
中图法分类号: TP391 **文献标识码**: A