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There Is No Minimal r.e. Degree in Every Nonzero [a]∈ R/M*

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Abstract: It is proved that given any nonrecursive r.e. degree **a**, there exist r.e. degrees $\mathbf{c}<\mathbf{a}$ and $\mathbf{d}\in\mathbf{M}$ such that $\mathbf{a}\leqslant\mathbf{d}\cup\mathbf{c}$. Therefore, there is no minimal r.e. degree in every nonzero $[\mathbf{a}]\in\mathbf{R}/\mathbf{M}$, the quotient upper semilattice of the recursively enumerable degrees modulo the cappable r.e. degrees, i.e., given any noncappable r.e. degree **a** there is an r.e. degree $\mathbf{c}<\mathbf{a}$ such that $[\mathbf{c}]=[\mathbf{a}]$.

Key words: Turing degree, recursively enumerable set, minimal pair

An r.e. degree **a** is *cappable* if **a** is a half of a minimal pair; otherwise, **a** is *noncappable*. Let **M** denote the class of all the cappable r.e. degrees together with **0**; let $\mathbf{NC}=\mathbf{R}-\mathbf{M}$ denote the class of all the noncappable r.e. degrees.

Ambos-Spies, Jockusch, Shore and Soare[1] proved that **M** is an ideal in **R**. Thus we have a quotient of the r.e. degrees **R** modulo the cappable degrees **M**, denoted by **R**/**M**. Elements in **R**/**M** are denoted by [**a**], the equivalence class of $\mathbf{a}\in\mathbf{R}$, i.e., $[\mathbf{a}]=\{\mathbf{b}\in\mathbf{R}:\mathbf{b}\sim\mathbf{a}\}$, where $\sim$ is an equivalence relation defined in **R** such that $\mathbf{a}\sim\mathbf{b}$ iff

$$\exists\ \mathbf{c}_1,\mathbf{c}_2\in\mathbf{M}(\mathbf{a}\cup\mathbf{c}_1=\mathbf{b}\cup\mathbf{c}_2).$$

Given any $[\mathbf{a}],[\mathbf{b}]\in\mathbf{R}/\mathbf{M}$, $[\mathbf{a}]\leq[\mathbf{b}]$ if there is an r.e. degree $\mathbf{c}\in\mathbf{M}$ such that $\mathbf{a}\leqslant\mathbf{b}\cup\mathbf{c}$. $[\mathbf{a}]<[\mathbf{b}]$ if $[\mathbf{a}]\leq[\mathbf{b}]$ and $[\mathbf{b}]\not\leq[\mathbf{a}]$. Let $[\mathbf{a}]\vee[\mathbf{b}]$ denote the least upper bound of [**a**] and [**b**]. It is easy to prove that **R**/**M** is an upper semilattice, and $[\mathbf{a}]\vee[\mathbf{b}]=[\mathbf{a}\cup\mathbf{b}]$. Schwarz[2] proved the downward density theorem in **R**/**M**. (Jockusch[3] commented that the downward density theorem in **R**/**M** follows directly from the Robinson's splitting theorem and the fact that $\mathbf{NC}=\mathbf{LCu}$, the set of all the r.e. degrees which cup to $\mathbf{0}'$ by a low r.e. degree.) In this paper we shall prove that given any nonrecursive r.e. degree **a** there exist r.e. degrees $\mathbf{c}<\mathbf{a}$ and $\mathbf{d}\in\mathbf{M}$ such that $\mathbf{a}\leqslant\mathbf{d}\cup\mathbf{c}$. Therefore, there is no minimal r.e. degree in every nonzero $[\mathbf{a}]\in\mathbf{R}/\mathbf{M}$, i.e., given any r.e. degree **a** such that $[\mathbf{a}]\neq[\mathbf{0}]$, there is an r.e. degree $\mathbf{c}<\mathbf{a}$ such that $[\mathbf{a}]=[\mathbf{c}]$.

Our notation is standard with a minor change, a reference is Soare[4]. A number x is *unused at stage* $s+1$ if

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$x \geqslant s$ is greater than any number mentioned so far in the construction. If the oracle is a join of two sets, we assume that the use is computed on the two sets separately, i.e., $\Gamma((A\oplus B)\lceil(\gamma(x)+1);x)=\Gamma(A\lceil(\gamma(x)+1)\ominus B\lceil(\gamma(x)+1);x)$, where $\gamma(x)$ is the use of $\Gamma(A\oplus B;x)$. All use functions are assumed to be increasing in argument and nondecreasing in the stages.

1 Main Theorem and Its Requirements

Theorem 1.1. Given any nonrecursive r.e. degree **a** there exist r.e. degrees $\mathbf{c}<\mathbf{a}$ and $\mathbf{d}\in\mathbf{M}$ such that $\mathbf{a}\leqslant\mathbf{d}\cup\mathbf{c}$.

Corollary 1.2. For any $[\mathbf{a}]\in\mathbf{R}/\mathbf{M}$ with $[\mathbf{0}]<[\mathbf{a}]$ there is no minimal r.e. degree in $[\mathbf{a}]$.

The proof of Theorem 1.1. Given any nonrecursive r.e. set A, we shall recursively construct sets C, D and a recursive functional Γ such that $C<_T A$; $D\in\mathbf{M}$; and $A=\Gamma(C\oplus D)$, namely, the construction will satisfy for every e the following requirements:

$$\begin{aligned}&\mathscr{R}_e: A\neq\Phi_e(C);\\&\mathscr{P}_e: E\neq\omega-W_e;\\&\mathscr{N}_e: \{e\}^D=\{e\}^E=f_e\ \text{total}\rightarrow f_e\leqslant_T\varnothing,\end{aligned}\tag{1.1}$$

where E is an auxiliary set to guarantee that $D\in\mathbf{M}$ by satisfying $\mathscr{P}_e$ and $\mathscr{N}_e$ for all $e\in\omega$.

To define Γ, let $\gamma_s(n)$ denote the use of $\Gamma_s(C_s\oplus D_s;n)$ at the end of stage s. We shall satisfy the following conditions: for all n and s,

$$n\in A_{s+1}-A_s\longleftrightarrow\gamma_s(n)\in C_{s+1}-C_s;\tag{1.2}$$

$$n<\gamma_s(n)\leqslant\gamma_{s+1}(n),\ \gamma_s(n)<\gamma_s(n+1);\tag{1.3}$$

$$\gamma_s(n)<\gamma_{s+1}(n)\rightarrow(C\oplus D)_s\lceil(\gamma_s(n)+1)\neq(C\oplus D)_{s+1}\lceil(\gamma_s(n)+1);\tag{1.4}$$

$$\gamma(n)=\lim_s\gamma_s(n)\downarrow.\tag{1.5}$$

Condition (1.2) guarantees that $C\leqslant_T A$. Conditions (1.2)~(1.5) guarantee that $A\leqslant_T C\oplus D$. Since $\gamma\leqslant_T C\oplus D$ by (1.4) and (1.5), for each n, if s is such that $(C\oplus D)\lceil(\gamma_s(n)+1)=(C\oplus D)_s\lceil(\gamma_s(n)+1)$ then $n\in A$ iff $n\in A_s$ by (1.2) and (1.4).

We attack the positive requirement $\mathscr{P}_e$ by a regular diagonal argument. That is, we attempt to find an x such that $x\in W_e$ and enumerate x in E.

The basic module for the negative requirement $\mathscr{N}_e$ is to guarantee that once $\{e\}_s^{D_s}(x)\lceil x=\{e\}_s^{E_s}\lceil x$ we preserve at least one side of these computations. Suppose that at some later stage $t>s$, we enumerate some element into D and destroy the D-side computations, i.e., $\{e\}_t^{D_t}\lceil x\neq\{e\}_s^{D_s}\lceil x$, then we preserve the E-side computations until some stage $t'>t$ when the D-side computations restore, i.e., $\{e\}_{t'}^{D_{t'}}\lceil x=\{e\}_s^{D_s}\lceil x(=\{e\}_s^{E_s}\lceil x)$. Only after stage t' shall we allow the E-side to change. If we succeed in doing so and $\{e\}^D=\{e\}^E=f_e$ then we can prove that $f_e\leqslant_T\varnothing$. At any stage $s+1$, we define *the length of agreement as follows*:

$$l^{\mathscr{N}}(e,s)=\max\{x:\forall\ y<x(\{e\}_s^{D_s}(y)=\{e\}_s^{E_s}(y)\downarrow)\}.$$

A stage s is e-expansionary if $l^{\mathscr{N}}(e,s)>l^{\mathscr{N}}(e,t)$ for every $t<s$. When $l^{\mathscr{N}}(e,s)$ reaches a new value then elements can be enumerated in D or E, but not both.

For a single requirement $\mathscr{R}_e$, at any stage $s+1$, we define *the length of agreement and restraint function*:

$$l^{\mathscr{R}}(e,s)=\max\{x:\forall\ y<x(A_s(y)=\Phi_{e,s}(C_s;y))\},$$

$$r^{\mathscr{R}}(e,s)=\max\{u(C_s;e,x,s):x\leqslant l^{\mathscr{R}}(e,s)\}.$$

We assign to $\mathscr{R}_e$ a witness z. At any stage $s+1$, if

$$r^{\mathscr{R}}(e,s)>\gamma_s(z),\tag{1.6}$$

then we attempt to enumerate $\gamma_s(z)$ in D, and move $\gamma_s(z)$ to an unused number.

While the strategies for requirements in isolation are very simple, there are obviously several conflicts between them. The main conflict between the strategies is as follows. To ensure that $A=\Gamma(C\oplus D)$ and $C\leqslant_T A$, at any stage $s+1$, if $n\in(A_{s+1}-A_s)$ for some n, we have to enumerate $\gamma_s(n)$ in C_{s+1}. But this action will be restrained by some $r^{\mathscr{R}}(e,s)$. This means that $r^{\mathscr{R}}(e,s)$ may be destroyed, since $\gamma_s(n)<r^{\mathscr{R}}(e,s)$. One way to cope with this is to enumerate $\gamma_s(n)$ in D if $\gamma_s(n)<r^{\mathscr{R}}(e,s)$, but a minimal pair strategy may be destroyed.

In the next section we shall define the priority tree and use the tree method to overcome the conflict.

2 Priority Tree and the Basic Module

The priority tree $T=\omega^{<\omega}$. We define an order $<_L$ on T as follows: for any $\alpha,\beta\in T$,

$$\alpha<_L\beta\longleftrightarrow\alpha\subset\beta\vee\exists\ \tau\subseteq\alpha,\beta\exists\ a,b\in\omega(a<b\ \&\ \tau^\frown a\subseteq\alpha\ \&\ \tau^\frown b\subseteq\beta).$$

A node α is *odd* if $|\alpha|$ is odd; otherwise, α is even. We assign $\mathscr{N}_e$ to α if $|\alpha|=e$; $\mathscr{R}_e$ to α if $|\alpha|=2e-1$; and assign $\mathscr{P}_e$ to α if $|\alpha|=2e$. We say that α is a *strategy for the requirements* assigned to it.

We define a sequence δ_s of *nodes accessible at stage* $s+1$ as follows. Let

$$l(\alpha,s)=l^{\mathscr{N}}(|\alpha|,s).$$

A stage s is an α-stage if $\alpha\subseteq\delta_s$ or $s=0$. A stage s is α-expansionary if $s=0$ or s is an α-stage and $l(\alpha,s)>\max\{l(\alpha,t):t<s\ \&\ \alpha\subseteq\delta_t\}$.

Now we define $\delta_s(n)$ by induction on n for $n<s$. Suppose that $\alpha=\delta_s\lceil n$. Then $\delta_s(n)=0$ if s is an α-expansionary stage, and $\delta_s(n)=1+$ the greatest α-expansionary stage $t<s$, otherwise. The *true path* δ is defined by

$$\delta=\lim_s\inf\delta_s,$$

the leftmost path on which every node is visited infinitely often. The true path δ exists, simply because for any α, if $\alpha^\frown a$ is accessible at some stage $s+1$ then $\alpha^\frown b$ for any $b>a$ is accessible at any stage $t+1>s+1$ only if there is an α-expansionary stage between $s+1$ and $t+1$.

During the construction some node $\alpha\in T$ will be initialized at certain stages. We say that $\alpha\in T$ is *initialized at stage* $s+1$ if every parameter associated with α is set to be undefined.

2.1 The $\mathscr{R}_e$-module

To satisfy $\mathscr{R}_e$, let α be a strategy for $\mathscr{R}_e$. At any α-stage s, if there is no witness of α then set $z_\alpha(s+1)$ to be the least unused number; otherwise, let $z_\alpha(s)$ be the witness of α at stage $s+1$, and if $r^{\mathscr{R}}(e,s)>\gamma_s(z_\alpha(s))$ then enumerate $\gamma_s(z_\alpha(s))$ in D, and move $\gamma_s(z_\alpha(s))$ to an unused number.

3 Construction

A strategy α for $\mathscr{R}_e$ requires attention at stage $s+1$ if $\alpha\subseteq\delta_s$, and $\gamma_s(z_\alpha(s))<r^{\mathscr{R}}(e,s)$. A strategy α for $\mathscr{P}_e$ requires attention at stage $s+1$ if $E_s\cap W_{e,s}=\varnothing$, $\alpha\subseteq\delta_s$ and

$$\exists\ x(x\in W_{e,s}\ \&\ x>\max\{t<s:\delta_t<_L\alpha\}).\tag{3.1}$$

Stage $s+1$: The construction will proceed by performing the following steps.

Step 1. Let $\gamma_{s-1}(s)$ be the least unused number.

Step 2. For every odd $\alpha\subseteq\delta_s$, if there is no witness of α then let $z_\alpha(s+1)$ be the least unused number.

Step 3. Find the least α such that α requires attention at stage $s+1$. If there is no such α then go to Step 4; otherwise, we say that α receives attention at stage $s+1$.

Substep 3.1. If α is a strategy for $\mathscr{P}_e$ then let x be least such that (3.1) holds, enumerate x in E, initialize every $\beta\supset\alpha$ and go to Step 4.

Substep 3.2. If α is a strategy for $\mathscr{R}_e$ then enumerate $\gamma_s(z_\alpha(s))$ in D, move $\gamma_s(n)$ for all n with $s\geqslant n\geqslant z_\alpha(s)$, maintaining their order, to the least unused numbers, initialize every $\beta\supset\alpha$ and go to Step 4.

Step 4. For any $n<s$, if $n\in(A_{s+1}-A_s)$ then enumerate $\gamma_s(n)$ into C and move $\gamma(m)$ for all m with $s\geqslant m\geqslant n$ to the least unused numbers.

Step 5. Initialize every $\beta>_L\delta_s$.

This ends the description of the construction.

4 Verification

Let

$$\delta=\lim_s\inf\delta_s$$

be the true path. We prove Lemmas 4.1～4.3 by induction on $\alpha\subset\delta$. Given any $\alpha\subset\delta$, assume that Lemmas 4.1～4.3 hold for every $\alpha'\subset\alpha$.

Lemma 4.1. If $\alpha\subset\delta$ is a strategy for $\mathscr{R}_e$ then $\mathscr{R}_e$ is satisfied eventually, and requires attention only finitely often.

Proof. Let s_0 be the least α-stage such that no $\beta<_L\alpha$ receives attention after stage s_0, $z_\alpha=\lim_s z_\alpha(s)\downarrow=z_\alpha(s_0)$ and $A_{s_0}\lceil(z_\alpha+1)=A\lceil(z_\alpha+1)$.

For the sake of a contradiction, assume that $\mathscr{R}_e$ is not satisfied. Then $A=\Phi_e(C)$ and $r^{\mathscr{R}}(e,s)$ tends to infinity. Hence, α requires attention at infinitely many α-stages s to enumerate $\gamma_s(z_\alpha)$ in D and move $\gamma_s(z_\alpha)$ to an unused number. We now show that $A\leqslant_T\varnothing$. To recursively decide whether $n\in A$ for any given $n\in\omega$, find an α-stage $s\geqslant s_0$ such that $l^{\mathscr{R}}(e,s)>n$ and $\gamma_s(z_\alpha)>u(C_s;e,n,s)$, then $n\in A$ iff $n\in A_s$. Otherwise, if $n\in(A-A_s)$ then $\Phi_{e,s}(C_s;n)=\Phi_e(C;n)=A(n)=A_s(n)$, a contradiction, since by the choice of s_0, $C_s\lceil\gamma_s(z_\alpha)=C\lceil\gamma_s(z_\alpha)$. Hence, A is recursive, a contradiction to the assumption that A is not recursive.

Therefore, $\mathscr{R}_e$ is satisfied. Let $p=\mu p(A(p)\neq\Phi_e(C;p))$. Then, if there is an α-stage $s\geqslant s_0$ such that $l^{\mathscr{R}}(e,s)=p$ and $\Phi_{e,s}(C_s;p)\downarrow$ then $r^{\mathscr{R}}(e,s)=\lim_s r^{\mathscr{R}}(e,s)$, since if $\gamma_s(z_\alpha)>r^{\mathscr{R}}(e,s)$ then $\Phi_{e,s}(C_s;p)=\Phi_e(C;p)$; if $\gamma_s(z_\alpha)<r^{\mathscr{R}}(e,s)$ then $\gamma_{s+1}(z_\alpha)>r^{\mathscr{R}}(e,s)$, and $\Phi_{e,s}(C_s;p)=\Phi_e(C;p)$. If there is no such α-stage s then $r^{\mathscr{R}}(e,s)=\lim_s r^{\mathscr{R}}(e,s)$ for any α-stage $s\geqslant s_0$ with $l^{\mathscr{R}}(e,s)=p$. Hence, α requires attention only finitely often.

Lemma 4.2. If $\alpha\subset\delta$ is a strategy for $\mathscr{P}_e$ then $\mathscr{P}_e$ is satisfied.

Proof. Let s_0 be the least stage such that no $\beta<_L\alpha$ receives attention after stage s_0. Assume that $\mathscr{P}_e$ is not satisfied, i.e., $W_e\cap E=\varnothing$. If there is no x such that $x\in W_e$ then $\mathscr{P}_e$ is satisfied, a contradiction; otherwise, let $x>s_0$ be a number such that $x\in W_e$, then at the least α-stage $s\geqslant s_0$ such that $x\in W_{e,s}$, α receives attention, x is enumerated in E at Substep 2.1 of stage $s+1$, and $\mathscr{P}_e$ is satisfied at $s+1$. A contradiction. Therefore, $\mathscr{P}_e$ is satisfied.

Lemma 4.3. If $\alpha\subset\delta$ is a strategy for $\mathscr{N}_e$ then $\mathscr{N}_e$ is satisfied.

Proof. Let s_0 be the least α-stage such that no $\beta<_L\alpha$ receives attention after stage s_0.

Assume that f_e is total. To recursively compute $f_e(x)$ for any given x, find an α-expansionary stage $s\geqslant s_0$ such that $l(\alpha,s)>x$, then

$$f_e(x)=\{e\}_s^{D_s}(x)=\{e\}_s^{E_s}(x)=p.$$

We prove by induction on t that for all $t\geqslant s$ either

$$\{e\}_t^{D_t}(x)=p,\text{ or}\tag{4.1}$$

$$\{e\}_t^{E_t}(x)=p.\tag{4.2}$$

Let $s=t_1<t_2<\ldots$ be the α-expansionary stages, then both (4.1) and (4.2) hold for t_1 since $t_1=s$. Fix any n and assume by induction that both (4.1) and (4.2) hold for $t\leqslant t_n$. Now at stage t_n+1, at most one element enters D or E, so at most one of the computations (4.1) and (4.2) for $t=t_n$ is destroyed. Now for any t with $t_n<t<t_{n+1}$, at stage $t+1$, there are two cases:

(a) If there is a β such that $\gamma_t(z_\beta(t))$ is enumerated in D at $t+1$ then we must have that

$$\gamma_t(z_\beta(t)) > \max\{u < t : \delta_u <_L \beta\}.$$

By the choice of s_0, $\beta \geqslant_L \alpha\hat{}1$, and $\beta \subseteq \delta_t$. Since $\alpha\hat{}0 \subseteq \delta_{t_n}$ and $\alpha\hat{}1 \leqslant_L \delta_t$, we have that $\delta_{t_n} <_L \beta$. Thus $\gamma_t(z_\beta(t)) > z_\beta(t) > t_n$.

(b) If some x is enumerated into E at stage $t+1$ by some β then $\alpha\hat{}1 \leqslant_L \beta$, $\beta \subset \delta_t$ and $x > \max\{u < t : \delta_u <_L \beta\}$. Since $\alpha\hat{}0 \subseteq \delta_{t_n} <_L \delta_t$, $\delta_{t_n} <_L \beta$, so $x > t_n$.

By (a) and (b) we know that no element $\leqslant t_n$ is enumerated into either E or D at the stage $t+1$ such that $t_n < t < t_{n+1}$. As the use of $\{e\}^{D}_{t_n}(x)$ and $\{e\}^{E}_{t_n}(x)$ is less than t_n we know that the computation (4.1) or (4.2) which is not injured at stage t_n+1 will be preserved until the next α-expansionary stage t_{n+1}. Thus we know that both (4.1) and (4.2) hold at stage $t_{n+1}+1$.

Lemma 4.4. $C \leqslant_T A$. By Lemma 4.1, we have that $C <_T A$.

Proof. By the construction, for every n, $\gamma_s(n)$ is enumerated in C at stage $s+1$ if and only if $A_s\lceil(n+1) \neq A_{s+1}\lceil(n+1)$. Hence, (1.2) is satisfied, and $C \leqslant_T A$, since $\gamma_s(n) > n$.

Lemma 4.5. $A = \Gamma(C \oplus D)$.

Proof. By the definition of $\gamma_t(n)$, (1.3) is satisfied.

Fix any n. By Lemma 4.1, there exist a strategy $\alpha \subset \delta$ and an α-stage s_0 such that α is not initialized after s_0, $z_\alpha = \lim_s z_\alpha(s) = z_\alpha(s_0) > n$ and $A_{s_0}\lceil(n+1) = A\lceil(n+1)$. Then $\lim_t \gamma_t(n) = \gamma_{s_0}(n)$, since $\gamma_t(n)$ moves at any stage $t+1 \geqslant s+1$ only if there is a strategy $\beta < \alpha$ such that β receives attention at stage $t+1$ or $A_t\lceil(n+1) \neq A_{t+1}\lceil(n+1)$. Hence, (1.5) is satisfied.

Since $\gamma_t(n)$ moves at any stage $t+1$ only if $\gamma_t(n)$ is enumerated in D or C, hence, (1.4) is satisfied.

This ends the proof of the theorem.

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每个非零的a∈R/M 中不存在极小元

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摘要: 证明了给定任何非零的递归可枚举图灵度a存在递归可枚举图灵度c<a和d∈M,使得a≤d∪c.由此可以得到:在每个非零[a]∈R∧M 中不存在极小元,即给定任何非可盖递归可枚举图灵度a,存在一个递归可枚举图灵度c<a,使得[c]=[a].

关键词: 图灵度;递归可枚举度;极小对

中图法分类号: TP301　　**文献标识码**: A