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一类基于小波框架的采样子空间*

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Sampling Subspaces Based on Wavelet Frames

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Abstract: Sampling theory is one of the most powerful results in modern information theory and technology. The digital signal with sampling properties can be reconstructed from its samples in a perfect form. Walter and Zhou extended the Shannon sampling theorem to wavelet subspaces. This paper improves the classical sampling theorems based on wavelet frames. A basic problem on information theory is introduced here. For a given digital signal, whether it has sampling series form. In this paper, the digital signals with sampling properties are characterized based on wavelet frames. For a given sapling subspace, the analytic form of the signals in it is proposed. Especially some new kinds of sampling subspaces are offered here. As an application, the examples show that the new theorems improve some known relating results, which is effective for the digital signals' sampling and reconstructions.
Key words: wavelet frame; sampling theorem; wavelet subspace; shift-invariant subspace; information reconstruction

摘 要: 采样定理是现代信息理论与技术的基本工具,具有采样性质的信号具有比较完美的信息重构形式.Walter 和 Zhou 将古典信号的采样定理发展到小波子空间.发展了基于小波框架的数字信号的采样定理与方法.提出并回答了信息理论中的一个基本问题:一个能量有限的数字信号,是否具有采样定理的形式.以小波框架为工具给出了具有采样性质的数字信号的刻画;对于一个给定的采样子空间,给出了该子空间的信号的表示形式;特别是给出了一大类新的具有采样性质的数字信号空间.应用实例表明,从理论和实践上对于数据信息的采样和重构是有意义的,是对以往相关结果的有效改进.
关键词: 小波框架;采样定理;小波子空间;平移不变子空间;信息重构

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1 Introduction

Sampling theory is one of the most powerful results in information theory and technology. The objective of sampling is to reconstruct a digital signal from its samples.

Notations: Firstly, we discuss functions in $L^2(\mathrm{R})$. Therefore $f=g$ means that $f(\omega)=g(\omega)$ for almost everywhere $\omega\in\mathrm{R}$.

For $\psi\in L(\mathrm{R})$, its Fourier transform is defined by $\widehat{\psi}(\xi)=\int_{\mathrm{R}}\psi(x)\mathrm{e}^{-2\pi ix\xi}\mathrm{d}x,\forall\xi\in\mathrm{R}$.

$\sum_n$ stands for summation over all $n\in\mathrm{Z}$.

$C(\mathrm{R})$ is the space of continuous function.

$L[-\frac{1}{2},\frac{1}{2}]=\{ f : f$ is 1-periodic and square integral on $L^2[-\frac{1}{2},\frac{1}{2}]\}$.

$G_f(\omega)=\sum_k\left|\widehat{f}(\omega+k)\right|^2$. It is easy to see that $G(\omega)$ is defined only a.e.

$E_f=\{\omega\in\mathrm{R}:G_f(\omega)>0\},\forall f\in L^2(\mathrm{R})$.

χ_E is the characteristic function of the set E.

$\widehat{f}^*=\sum_n f(n)e^{-i2\pi\omega}$ for $f\in L^2(\mathrm{R})$ with $\sum_n|f(n)|^2<\infty$.

$V_f^0=\{g:g(\cdot)=\sum_n c_n f(\cdot-n)$, where the convergence is in $L^2(\mathrm{R})$, $\{c_n\}_{n\in\mathrm{Z}}\in l^2\}$.

$V_f=\overline{\{f(\cdot-n)\}_n}$ which means that any $g\in V_f$ can be approximated arbitrarily well in norm by a finite linear combinations of vectors $f(\cdot-n)$, and V_f is called a shift invariant subspace generated by f.

$\mu(\cdot)$ is Lebesgue measure on R.

For the sets $A,B\subseteq\mathrm{R}$, $A\overset{a.e}{=}B$ means that $\mu((A-B)\cup(B-A))=0$, and $A\overset{a.e}{\subseteq}B$ means that $\mu(A-B)=0$.

A family $\{f_\lambda\}_{\lambda\in\Lambda}$ of elements in a Hilbert space H is called a frame for H, where Λ is a countable set, if there exist constants $L,M>0$ such that

$$L\|f\|^2\le\sum_{\lambda\in\Lambda}|<f,f_\lambda>|^2\le M\|f\|^2,\forall f\in\mathrm{H}.$$

The above constants L and M are called the frame bounds. If $L=M=1$, then the frame is called a tight frame.

For $\phi\in L^2(\mathrm{R})$, if $\{\phi(\cdot-n)\}_n$ is a frame (Riesz basis) for V_ϕ then ϕ is called a frame (Riesz) function.

Walter[1] extended the Shannon sampling theorem to wavelet subspaces. Zhou and Sun[2] characterized the general shifted wavelet subspaces on which the sampling theorem holds:

Proposition 1.[2] Suppose that $\phi\in L^2(\mathrm{R})$ and ϕ is a frame function. Then the following two assertions are equivalent:

(i) $\sum_k c_k\phi(\cdot-k)$ converges pointwisely to a continuous function for any $\{c_k\}\in l^2$ and there is a frame $\{\psi(\cdot-k)\}_{k\in\mathrm{Z}}$ for V_ϕ such that $f(x)=\sum_k f(k)\psi(x-k),\quad\forall f\in V_f$,

where the convergence is both in $L^2(\mathrm{R})$ and uniform on R.

(ii) ϕ is continuous, $\sum_k|\phi(\cdot-k)|^2$ is bounded on R and $A\chi_{E_\phi}(\omega)\le\left|\widehat{\phi}^*(\omega)\right|\le B\chi_{E_\phi}(\omega)$, *a.e.* for some constants $A,B>0$.

Moreover, it implies that for ϕ in (i), $f(k)=<f,\tilde{\phi}(\cdot-k)>$ for any $f\in V_\phi$, where $\tilde{\phi}$ is defined in the following Proposition 2.

For a given digital signal with finite energy, whether it has sampling series form. In this paper we will improve the classical sampling theorems based on wavelet frames. The digital signals with sampling properties are characterized based on wavelet frames. For a given sapling subspace, the analytic form of the signals in it is proposed. Especially some new kinds of sampling subspaces are offered here. The examples, as an application, show that the new theorems improve some known relating results, which is effective for the digital signals' sampling and reconstructions.

2 Definitions and Main Results

Definition 1. A closed subspace V in $L^2(\mathrm{R})$ is called a sampling space, if there is a frame $\{\psi(\cdot-k)\}_{k\in\mathrm{Z}}$ for V such that $\sum_k c_k\psi(x-k)$ converges pointwisely to a continuous function for any $\{c_k\}\in l^2$ and $f(x)=\sum_k f(k)\psi(x-k),\quad \forall f\in V$, where the convergence is both in $L^2(\mathrm{R})$ and uniform on R.

In this case, ψ is called a sampling function on V.

From the definition, we know that if V is a sampling space then for any $f\in V$ there exists a function $g\in C(\mathrm{R})$ such that $f(x)=g(x), a.e. x\in\mathrm{R}$. Therefore, in what follows we assume that all the functions in a sampling space are continuous.

Definition 2. Assume $\phi\in L^2(\mathrm{R})$. If $\sum_k c_k\phi(x-k)$ converges pointwisely to a continuous function for any $\{c_k\}\in l^2$, then ϕ is called a P-function.

Next we define two kinds of special functions to characterize the functions with sampling property.

Definition 3. f is called a P_1-function if the following conditions holds:

(i) $f\neq 0, f\in L^2(\mathrm{R})\cap C(\mathrm{R})$, $\sum_k |f(k)|^2<\infty$ and f is a bounded function;

(ii) $\{\omega:\hat{f}^*(\omega)=0\}\overset{a.e.}{\subseteq}\{\omega:G_f(\omega)=0\}$;

(iii) There exist two positive constants A,B such that $0<A\le\dfrac{G_f(\omega)}{|\hat{f}^*(\omega)|^2}\le B<\infty$ for almost everywhere $\omega\in E_f$.

Moreover, if f is a P_1-function and $\sup_{x\in\mathrm{R}}\sum_k|f(x-n)|^2<\infty$, then f is called a P_2-function. For any $f\in P_1$, it is easy to see that $\mu(E_f)>0$ and $\hat{f}^*(\omega)$, $G_f(\omega)$ are well defined almost everywhere.

For giving our results, firstly we list the following propositions.

Proposition 2[3,5]. Suppose $\phi\in L^2(\mathrm{R})$. ϕ is a frame function if and only if there are constants $A,B>0$ such that $A\chi_{E_\phi}\le G_\phi\le B\chi_{E_\phi}$, *a.e.*

Especially, if ϕ is a Riesz function, then $E_\phi=\mathrm{R}$.

Proposition 3[2,3]. Suppose that $f\in L^2(\mathrm{R})$ and f is a frame function. If

$$\hat{\tilde{f}}(\omega)=\begin{cases}\dfrac{\hat{f}(\omega)}{G_f(\omega)}, & \text{if } \omega\in E_f\\ 0, & \text{if } \omega\notin E_f\end{cases},$$

then $\{\tilde{f}(\cdot-n)\}_n$ is a dual frame of $\{f(\cdot-n)\}_n$ in V_f.

Proposition 4[3]. Assume that $\phi\in L^2(\mathrm{R})$ and $\phi\neq 0$. Define the function ϕ_0 via its Fourier transform by

$$\widehat{\phi_0}(\omega) := \begin{cases} \widehat{\phi}(\omega)\Phi^{-1/2}(\omega), & \text{if } \Phi(\omega) \neq 0 \\ 0, & \text{if } \Phi(\omega) = 0 \end{cases},$$

where $\Phi(\omega) = \sum_{k\in Z}\left|\widehat{\phi}(\omega+k)\right|^2$. Then $\{\phi_0(\cdot-n)\}_n$ is a tight frame for V_ϕ.

By Lemma 1 in [2], we have

Lemma 1. Suppose $\phi \in L^2(\mathrm{R})$. Then ϕ is a P-function if and only if the following holds.

(i) $\phi \in C(\mathrm{R})$,

(ii) $\sum_k |\phi(x-k)|^2 \le M < \infty$ for some constant M.

First, we have the following Lemma.

Lemma 2. Assume that ϕ is a frame function. If ϕ is a P-function, then for any function $f \in V_\phi$ with $\widehat{f}(\omega) = b(\omega)\widehat{\phi}(\omega)$ where $b(\omega)$ is a function with period 1 and bounded on E_ϕ, then f is also a P-function. Specially, for any frame function $\psi \in V_\phi$, ψ is a P-function.

Proof. Assume that $f \in V$ such that $\widehat{f}(\omega) = b(\omega)\widehat{\phi}(\omega)$ where $b(\omega)$ is a function with period 1 and bounded on E_ϕ. Let $B(\omega) = b(\omega)\chi_{E_\phi}(\omega)$. Since $B(\omega)$ is bounded on $[-\frac{1}{2},\frac{1}{2}]$, there exists $\{B_n\} \in l^2$ such that $B(\omega) = \sum_n B_n e^{-i2\pi n\omega}$.

Since ϕ is a P-function and $\widehat{f}(\omega) = b(\omega)\widehat{\phi}(\omega) = B(\omega)\widehat{\phi}(\omega)$, we have $f \in C(\mathrm{R})$ and $f(x) = \sum_n B_n\phi(x-n)$, where the convergence is both in $L^2(\mathrm{R})$ and pointwisely.

Since ϕ is a P-function, it is from Lemma 1 that $\sup_x \sum_n |\phi(x-n)|^2 < \infty$. Hence

$$\left.\begin{aligned} \sup_x \sum_k |f(x-k)|^2 &= \sup_x \sum_k \left|\sum_n B_n\phi(x-k-n)\right|^2 \\ &= \sup_x \int_{-1/2}^{1/2} |B(\omega)|^2 \left|\sum_k \phi(x-k)\mathrm{e}^{-i2\pi\omega}\right|^2 d\omega \\ &\le \sup_x \|B(\omega)\|_\infty^2 \sum_k |\phi(x-k)|^2 < \infty \end{aligned}\right\} \tag{2.1}$$

Since $f \in C(\mathrm{R})$, it is from (2.1) and Lemma 1 that f is a P-function.

For any frame function $\psi \in V_\phi$, there exists a function τ with period 1 such that $\widehat{\psi}(\omega) = \tau(\omega)\widehat{\phi}(\omega)$, then $G_\psi(\omega) = |\tau(\omega)|^2 G_\phi(\omega)$. By Proposition 2, τ is bounded on E_ϕ. Thus ψ is a P-function.

This completes the proof of Lemma 2.

The following propositions characterize the Shift-invariant subspaces.

Proposition 5[5]. $V_f = \{ g \in L^2(\mathrm{R}) : \widehat{g} = \tau\widehat{f}, \tau$ is a function with period 1, $\tau\widehat{f} \in L^2(\mathrm{R}) \}$.

Proposition 6[5]. () Suppose $g \in V_f$. $V_g = V_f$ if and only if $\operatorname{supp}\widehat{f} \overset{a.e}{=} \operatorname{supp}\widehat{g}$.

By Proposition 5, we have the following two Lemmas.

Lemma 3. Let $f, g \in L^2(\mathrm{R})$. If $V_f = V_g$ then $E_f \overset{a.e}{=} E_g$.

Lemma 4. If $f \in V_g$, then $V_f \subseteq V_g$.

Further we have

Lemma 5. Suppose f is a P_2-function. Then

$$\{\omega:\hat{f}^*(\omega)=0\}\overset{a.e}{=}\{\omega:G_f(\omega)=0\}\overset{a.e}{\subset}\{\omega:\hat{f}(\omega)=0\} \tag{2.2}$$

Proof. Let $C(\omega)=1-\chi_{E_f}(\omega)$. Then $C(\omega)\in L^2[-\frac{1}{2},\frac{1}{2}]$ and there exists $\{c_k\}\in l^2$ such that $C(\omega)=\sum_k c_k \mathrm{e}^{-i2\pi k\omega}$. By Lemma 1, $\sum_k c_k f(x-k)$ converges pointwisely to a continuous function. Since $C(\omega)\hat{f}(\omega)=0$, $\sum_k c_k f(x-k)$ =0 for any $x\in \mathrm{R}$. Then

$$\int_{[-\frac{1}{2},\frac{1}{2}]\setminus E_f}\left|\sum_n f(n)\mathrm{e}^{-i2\pi n\omega}\right|^2\mathrm{d}\omega=\int_{[-\frac{1}{2},\frac{1}{2}]}|C(\omega)|^2\left|\sum_n f(n)\mathrm{e}^{-i2\pi n\omega}\right|^2 d\omega=\sum_n\left|\sum_k c_k f(n-k)\right|^2=0.$$

Hence, $\{\omega:G_f(\omega)=0\}\overset{a.e}{\subset}\{\omega:\hat{f}(\omega)=0\}$. Since $f\in P$,we have $\{\omega:\hat{f}^*(\omega)=0\}\overset{a.e}{=}\{\omega:G_f(\omega)=0\}$. Then we have (2.2).

This completes the proof of Lemma 5.

By Lemma 3, if f is a Riesz function, then any frame function for V_f is a Riesz function.

Proposition 7[9]. Assume f is a P_1-function. Let

$$\hat{f}_p(\omega)=\begin{cases}\dfrac{\hat{f}(\omega)}{\hat{f}^*(\omega)}, & \text{if } \hat{f}^*(\omega)\neq 0\\ 0, & \text{if } \hat{f}^*(\omega)=0\end{cases},$$

then $\{f_p(\cdot-n)\}_n$ is a frame for V_f.

Proposition 8[9]. Assume that V is a shift invariant subspace. Then the following assertions are equivalent:

(i) V is a sampling space.

(ii) If ϕ is continuous and $\{\phi(\cdot-n)\}_n$ is a frame function for V, then $\sup_x\sum_n|\phi(x-n)|^2<\infty$ and there exist positive constants A,B such that

$$A\chi_{E_\phi}(\omega)\le\left|\hat{\phi}^*(\omega)\right|\le B\chi_{E_\phi}(\omega), a.e.$$

(iii) There exists a continuous function ϕ which is a frame function for V, such that $\sup_x\sum_n|\phi(x-n)|^2<\infty$, and $A\chi_{E_\phi}(\omega)\le\left|\hat{\phi}^*(\omega)\right|\le B\chi_{E_\phi}(\omega), a.e.$ for some positive constants A,B.

(iv) There exists a continuous function ϕ which is a frame function for V, such that $\sup_x\sum_n|\phi(x-n)|^2<\infty$ and $A\|g\|^2\le\sum_n|g(n)|^2\le B\|g\|^2, \forall g\in V$ for some positive constants A,B.

(v) There exists a continuous function ϕ which is a frame function for V, such that

$$\sup_x\sum_n|\phi(x-n)|^2<\infty \text{ and } \{\sum_l\overline{\phi(n-l)}\phi(n-\cdot)\}_n \text{ is a frame for } V.$$

Moreover, for Proposition 8 we have the following assertions.

If (i) holds and S is a sampling function, then

$$f(n)=\langle f,\tilde{S}(\cdot-n)\rangle, \forall n\in \mathrm{Z}, \forall f\in V.$$

If (ii) or (iii) holds, then the sampling function in (i) can be taken as S, where

$$\hat{S}(\omega)=\begin{cases}\dfrac{\hat{\phi}(\omega)}{\hat{\phi}^*(\omega)}, & \text{if } \omega\in E_\phi\\ 0, & \text{if } \omega\notin E_\phi\end{cases},$$

If (v) holds, then the sampling function in (i) can be taken as $\tilde{S}$, where $S(\cdot)=\sum_l\overline{\phi(-l)}\tilde{\phi}(-\cdot)\}_n$.

Next, we give a necessary condition for a function f to be in a sampling space.

Theorem 1. Assume that V is a sampling space and ψ is a sampling function for V. Let $f \neq 0$ and $f \in V$. Then f is a P_1-function. Moreover, if $\hat{f}(\omega) = b(\omega)\hat{\psi}(\omega)$ where $b(\omega)$ is a function with period 1 and bounded on E, then f is a P_2-function. Specially, any frame function ϕ in V is a P_2-function.

Proof. Assume that $f \neq 0, f \in V$ and V is a sampling space with sampling function S. By Proposition 8, we have $f \in C(\mathrm{R})$, $\sum_k |f(k)|^2 < \infty$ and $f(x) = \sum_k f(k)S(x-k)$. Since

$$|f(x)|^2 \leq \sum_k |f(k)|^2 \sum_k |S(x-k)|^2 < \infty, \forall x \in \mathrm{R},$$

f is a bounded function.

Take Fourier transformation for $f(x) = \sum_k f(k)S(x-k)$, we have

$$\hat{f}(\omega) = \hat{f}^*(\omega)\hat{\psi}(\omega), \quad G_f(\omega) = \left|\hat{f}^*(\omega)\right|^2 G_\psi(\omega).$$

Thus

$$\{\omega : \hat{f}^*(\omega) = 0\} \overset{a.e}{\subseteq} \{\omega : G_f(\omega) = 0\}, \tag{2.3}$$

$$E_f \subseteq E_\psi.$$

Since ψ is a frame function for V, it is form Proposition 2 that there exist positive constants A, B such that

$$A \leq \frac{G_f(\omega)}{\left|\hat{f}^*(\omega)\right|^2} = G_\psi(\omega) \leq B, \ a.e. \ E_f. \tag{2.4}$$

By (2.3)) and (2.4), f is a P_1-function.

Assume that $f \in V$ such that $\hat{f}(\omega) = b(\omega)\hat{\psi}(\omega)$ where $b(\omega)$ is a function with period 1 and bounded on E_ψ. Since ψ is a P-function, it is form Lemma 2, f is also a P-function. By Lemma 1,

$$\sup_x \sum_n |f(x-n)|^2 < \infty \tag{2.5}$$

By (2.5) and f is a P_1-function, we get f is a P_2-function.

For any frame function $\phi \in V$, there exists a function b with period 1 such that $\hat{\phi}(\omega) = b(\omega)\hat{\psi}(\omega)$, then $G_\phi(\omega) = |b(\omega)|^2 G_\psi(\omega)$. By Proposition 2, b is bounded on E_ψ. Thus ϕ is a P_2-function.

This completes the proof of Theorem 1.

Next, we give a kind of sampling space.

Theorem 2. Assume that $\sum_n |f(\omega+n)| \in L[-\frac{1}{2},\frac{1}{2}] \cap L^2[-\frac{1}{2},\frac{1}{2}]$ and there exists $E \subseteq [-\frac{1}{2},\frac{1}{2}]$ such that

$$A_1\chi_E(\omega) \leq \sum_n |f(\omega+n)|^2 \leq B_1\chi_E(\omega), \ a.e. \ \omega \in [-\frac{1}{2},\frac{1}{2}] \tag{2.6}$$

$$A_2\chi_E(\omega) \leq \sum_n |f(\omega+n)|^2 \leq B_2\chi_E(\omega), \ a.e. \ \omega \in [-\frac{1}{2},\frac{1}{2}] \tag{2.7}$$

where $A_i, B_i \ (i = 1,2)$ are positive constants. Then $f \in L(\mathrm{R}) \cap L^2(\mathrm{R})$. Let

$$\mathrm{g}(\xi) = \int_{\mathrm{R}} f(x)e^{i2\pi x\xi} d\xi, \forall \xi \in \mathrm{R}.$$

Then V_g is a sampling space with frame function g.

Proof. Note that $\sum_n |f(\omega+n)| \in L[-\frac{1}{2},\frac{1}{2}]$ if and only if $f \in L(\mathrm{R})$. Then $g \in C(\mathrm{R})$. By (2.6), we get $f \in L^2(\mathrm{R})$ and $f = \hat{g}$. By Proposition 2, g is a frame for V_g.

Since $\sum_n f(\omega+n)e^{i2\pi x(\omega+n)} \in L^2[-\frac{1}{2},\frac{1}{2}]$ and

$$g(x+k)=\int_{R} f(\omega)e^{i2\pi x\omega}e^{i2\pi k\omega}d\omega=\int_{-1/2}^{1/2}\sum_n f(\omega+n)e^{i2\pi x(\omega+n)}e^{i2\pi k\omega}d\omega, \tag{2.8}$$

we have

$$\sum_n |g(x+k)|^2=\int_{-1/2}^{1/2}\left|\sum_n f(\omega+n)e^{i2\pi x(\omega+n)}\right|^2 d\omega \le \int_{-1/2}^{1/2}\left|\sum_n |f(\omega+n)|\right|^2 d\omega \overset{\Delta}{=} B \tag{2.9}$$

Since $\sum_n f(\omega+n)\in L^2[-\frac{1}{2},\frac{1}{2}]$, it is from (2.8) that we have

$$\sum_n f(\omega+n)=\sum_k g(k)e^{-i2\pi k\omega}, a.e.$$

By (2.7), we have

$$A_2\chi_E(\omega)\le\left|\sum_k g(k)e^{-i2\pi k\omega}\right|\le B_2\chi_E(\omega), a.e.\ \omega\in[-\frac{1}{2},\frac{1}{2}] \tag{2.10}$$

Note that g is a frame for V_g. Since (2.9) and (2.10), it is from Proposition 8 that V_g is sampling space with frame g.

This completes the proof of Theorem 2.

3 Applications

Example 1. Let $\hat{f}(\omega)=\chi_E(\omega)$, where E is a measurable set in $[-\frac{1}{2},\frac{1}{2}]$. By Theorem 2, V_f is a sampling space with sampling function f and $V_f=\{g\in L^2(R):\mathrm{supp}\,\hat{g}\subseteq E\}$.

Similarly, let $\hat{g}(\omega)=\left(\frac{\sin\frac{\omega}{2}}{\frac{\omega}{2}}\right)^{2n}$, V_g is a sampling space with sampling function g.

Example 2. Let $\Omega>\frac{1}{2}$ and $A,B>0$. Assume that $g\in L^2(R)$, $E=\mathrm{supp}\,\hat{g}\subseteq[-\Omega,\Omega]$ with $\mu(E)>0$, and $A\chi_E(\omega)\le\hat{g}(\omega)\le B\chi_E(\omega)$. In this case, if we let $f=\breve{g}$, then f satisfies the conditions in Theorem 2. Therefore V_g is a sampling space with frame function g.

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References:

[1] Walter G. A sampling theorem for wavelet subspaces. IEEE Trans. on Inform. Theory, 1992,38(2):881−884.

[2] Zhou XW, Sun WC. On the sampling theorem for wavelet subspace. J. Fourier Anal. Appl., 1999,5(4):347−354.

[3] Boor C, Devore R, Ron A. The structure of finitely generated shift-invariant subspaces in $L^2(R)$. J. Func. Anal., 1994,119(1):37−78.

[4] Boor C, Devore R, Ron A. Approximation from shift-invariant subspaces of $L^2(R)$. Trans. Amer. Math, Soc., 1994,341:787−806.

[5] Bownik M. The structure of shift-invariant subspace of $L^2(R^n)$. J. Func. Anal., 2000,177(2):282−309.

[6] Ron A, Shen ZW. Frames and stable bases for shift-invariant subspaces of $L^2(R^d)$. Can. J. Math., 1995,5:1051−1094.

[7] Christensen O. An Introduction to Frames and Riesz Bases. Boston: Birkhauser, 2003.

[8] Zhao C, Zhao P. Sampling theorem and irregular sampling theorem for multiwavelet subspaces. IEEE Trans. on Signal Processing, 2005,53(2):705−713.

[9] Li XZ, Yang DY. A further characterization on the sampling theorem for wavelet subspaces. In: Computational Science-ICCS. LNCS 4488, 2007. 1021−1028.

[10] Yang DY, Zhou XW, Yuan ZZ. Frame wavelets with compact supports for $L^2(R^n)$. Acta Mathematica Sinica, English Series, 2007, 23(2):349−356.

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